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THE PHILOSOPHY OF POSSIBILITY

NO INSISTENCE has been more characteristic of modern philosophy than that which lays it down that philosophy has to do with experience and its actualities. Possibilities have been too often treated as airy nothings, mere ideas or mental terms, and nothing more. That philosophy is the science of the possible is a suggestion far from likely to be taken up even if it were found feasible or desirable, so insistent are the demands of the actual world for explanation. If we should find the actual to be the source of all possibility, that would certainly not render the suggestion, with its isolation of the possible, a more warrantable enterprise. But, in any case, the philosophy of possibility seems to me worthy of more attention than it has received. In spite of pure empiricism, possibility has not failed to catch some attention, all through the history of philosophy from the Greeks onward, however fugitive and sporadic that attention may have been. And so far from being a mere figment of the imagination, possibility exists both as idea and as fact; a possibility contains some actual idea; it is indeed as real, objectively. as actuality itself. But yet the possible implies in every case, that the idea has fallen short of reality. For every possibility an *ens essentiae* may be claimed, though not, of course, an *ens existentiae*. Every possibility has a real foundation in some nature or being, proximate or remote. That there is some reality in possibilities must be thus early kept in

mind. There are two forms of existence, the actual and the possible. What exists as actuality is, from the subject's point of view, content of a presentation, or reducible to such. Actual experience has a large margin of possible experience, and this extension of the actual cannot be left out of sight. There is not one of the States of America which does not exist for me actually, and not as mere possibility, and yet any one of these is only possible experience for me until I set foot upon it, and it becomes for me actual in presentative experience. What exists as possibility, on the other hand, is, from the subject's point of view, content of a conception. Such conceived content exists only as possibility, not actuality. Possibility has been declared to be just the ideal we have of anything. Possible existence may be of a kind, as we have just seen, that it can become real content of what is as yet only possible experience. But that is not the sense of possibility with which we are at this point concerned. We are concerned with it only in the sense in which the existent is present to consciousness purely as possible. The purely possible, of course, precludes existence. Possible being is not yet existent, but is taken as capable of coming into being, or existing. The idea of possibility exists antecedently to all created being. All knowing is a knowing of what is at least possible existence. All really possible objects are conceivable: all real possibilities are rational, I mean as objects of thought: the impossible is self-contradictory or irrational. The possible, in the logical sense, is what is free from contradiction; but all possibility is possibility of something, however indeterminate. The philosophy of possibility cannot evade the question of the origin of possibilities. Can we trace possibility simply to the human mind? Do possibilities not exist before the human mind comes into being? Will the possibilities not exist after the human mind has ceased to exist? Can we even ascribe the possibilities to

the universe? If the universe were done away, would possibilities not remain in undiminished form? For, are the possible universes not infinite? And, is not possibility necessary and eternal? These are among the questions that may be asked. The philosophy of possibility can hardly be satisfied to accept possibilities as accounting for themselves. We are compelled to think of the ideas of possibility as existing in some mind or spirit, and ultimately, in a Sovereign Mind or Spirit, wherein they gain eternal basis and fixity. But, even if such an ultimate origin be deemed unnecessary, it still holds that necessary ideas of possibility, like other necessary truths, contain "the determining plan and the regulative principle of existent things themselves." But if the ideas or principles exist before contingent things in this manner, then must they be grounded in some necessarily existing substance. Rosmini, who was severely critical of Kant's treatment of the categories, did possibility the honor to regard it as the only one, out of Kant's twelve forms, which really is an original and essential form of the human intellect. Höffding, less correctly, in my judgment, would educe all the categories to the two concepts, quantity and cause. Rosmini's position would perhaps be a primary consideration in making philosophy a science of the possible, were such a philosophy feasible. Our knowledge, so far at least as it consists of thought, should then only be concerned with possibilities. Possibility would be the fundamental, all-embracing category. We cannot, however, carry out this notion of confining all that is thought only to the possible, for we must know the real, and seek objective being or existence. Otherwise there would be a false limitation of thought. Besides, being allows itself to be brought under concepts, and thereby shows itself to be logically determined. But this is not all that Rosmini did. Aristotle had confused the purely logical issue, so far as it was concerned, by plac-

ing possibility and necessity in things, in conformity with his insistences on potentiality and actuality. But we shall consider Aristotle later. The transition from the possible to the actual was, for him, effected through motion. Kant did not improve matters when he made possibility and necessity mere subjective conditions of the thinking subject. What Rosmini did was to ground both possibility and necessity in the nature of being. Being so meant has no material associations, is only indeterminate form. But being, of course, is object, not subject. What I have already said of possibilities, from the subject's point of view, holds of them as known in their groundedness in being. But that is not to say that a merely possible essence has no ontological or objective basis independently of our conception of it. Possibility and necessity I have coupled together, since, possibility being analyzed, what is really possible is found to be necessarily so. And it may be remarked that it is with necessary truths that science, in the strict sense, has to do—with essences rather than existences. Mere scientific facts are sterile until they become fecundated by ideal or necessary truths.

When we consider the concept of mere possibility, we find that, in its positive aspect of capability of existence, it has a certain ontological basis and a certain objective reality, that is, as an objective concept. For what is meant by the really possible? Is it not some ideally constituted being or entity, conceived as capable of coming into being or existence in the world? But that means that it is already real as an objective concept. And this means that possible essence is not any fanciful and arbitrary creation which one chooses to conceive. Possible existence lends itself to no such freaks of the mind. Possible being, as a concept, needs some sort of objective reality to justify it. Possible existence is not to be thought of save as capable of being produced in the order of things existing; for

that alone is meant by its real possibility. Thus an acorn may, certain ideal conditions being made actual, be an oak, but it cannot possibly be an elm. In the same way the child may possibly be the man, but not the kangaroo. Possible essence does not require to be precontinued in the entity which is the real basis of possibility, in any formal and actual manner: it is enough that it virtually or ideally so pre-exists. Possible things are such that they may yet exist *de facto,* and do presently exist virtually in their causes. What has been said in this paragraph appertains to the region of metaphysical possibility, rather than that of logical possibility, although the mental process of conceiving must, of course, be of a logical or consistent character. But indeed this is not all; for, metaphysical as may be the entity which is the real basis of possibility, the possible essence must obviously be logically pre-contained in that existence which is the only basis of its reality.

The Scholastic philosophers have been by no means alone in realizing the importance of the philosophy of possibility, but they have been paramount in the attention they have given to it, and have made us their doctors for all time. One of the distinctions whose importance they have realized is that between intrinsic possibility and extrinsic possibility. In intrinsic possibility, the conception of a being or object, in its capacity for existence, is such as to involve no inner contradiction, no inner repugnance of being or character. Such intrinsic possibility is, in its inherent character, logical; yet must it find foundation in some reality.

A very broad example of intrinsic impossibility may be found in Lotze's criticism of Kant's scheme of Categories when, in the first volume of his "Metaphysics," he says that "that kind of theoretical security for an unconditional completeness, which Kant was in quest of, is something intrinsically impossible." But simpler examples, such as a round

square, lie ready to hand. The absolute character of inner or intrinsic possibility is noteworthy, compared with the relative character of outer or extrinsic possibility. But the two forms together constitute the full concept of possibility. And the inner possibility is the presupposition of the outer possibility. Such inner possibility is in character metaphysical. What is intrinsically impossible is also extrinsically impossible, but what is, in a given case, extrinsically impossible, is not therefore intrinsically impossible. Intrinsic possibility, in briefest terms, then, means mere non-absurdity.

Extrinsic possibility denotes the capacity for existence of a being or object due to the fact that something else has power to actualize it, as intrinsically possible. More briefly, extrinsic possibility means merely, being causable. Within the sphere of created things are many intrinsically possible things which are yet impossible, extrinsically. Extrinsic possibility may be physical, or it may be moral. Ultimate possibilities belong to the order of the necessary and immutable. Aristotle held, in his "Metaphysics," that the actual is anterior to the possible, alike in respect of being and of knowledge.

Aristotle contended that the passage from possibility to actuality takes place in certain fixed and unchanging ways, whereby the true nature of the real is made manifest. The potential is not to be confounded with mere possibility, as if anything whatsoever were to be reckoned possible. The possibility of the actual is for him the only possibility. What cannot be actualized is impossible. Determinate possibility, not possibility of the abstract and unlimited sort, is Aristotle's insistence. That is to say, the possible means possibility of realization in certain and definite ways. The impossible means incompatibility with the actual. In these positions the Scholastic philosophers have largely followed him. They have held that actual being, not possible, is the

first of all being. Their position has been that we cannot know possibility or things possible, without the concept of actuality or of things actual. Thus the concept of the possible presupposes the concept of the actual. The possible, is already, in part, the real. It has had to be admitted, of course, that in the case of secondary created existences or things, the idea of their possibility preceded their actual being. But, it is said, the contention as to the possible is not made in the sense of comparing it with the real in the same object. What is meant is, that a possible thing does not, in becoming real, give itself reality, its reality not being attained save through some other being, actual or existing.

Hegel puts the matter rightly when he says that "possibility should come second"—after actuality; for only "in abstract thought" does the possibility conception come first.

From the Scholastic philosophers I pass to that admittedly profound but neglected thinker, C. H. Weisse, who went to the root of the philosophy of possibility by raising —but not for the first time—the question of the possibility of God. The concept of God was for Weisse no presuppositionless affair. The original possibility of God includes for him every other possibility, and is the sole content of thought-necessity. This thought of the original possibility of God is the basal thought of his system, in which the concept of the possible may be said to count for more than the concept of being. But he held that no abstract necessity of reason can give us more than empty forms of possibility. His position, then, is that there is only one truth which is originally necessary to thought, namely, that only God is possible, and that in His possibility is contained the possibility of all things. In his view, the becoming real of this original possibility or thought-necessity of God means a real thinking absolute Subject. Now, interesting as these positions of Weisse are, they tempt one to some

critical reflections. The idea of any being implies, of course, the possibility of that being. But can we apply that to the case of God? Is not such mere possibility excluded by the fact that the true idea of God is that of the necessary being or existence? If He is necessarily the primal and perfect actuality, raised above all conditions of potentiality, then He is not possible in the sense that was spoken of. Kant, it should be noted, had early taken up the ground that God's existence, as a necessary Being, was antecedent to the possibility of His existence—the possibility, in his view, depending on His existence. What Kant really meant was, that possibility logically presupposes actual existence as its base or foundation, but his mode of putting the matter was not very happy. But Kant put the matter much better when, in his Critique of Judgment, he spoke of "the irrepressible tendency of reason to suppose some unconditionally necessary existence, or original ground, in which the distinction of possible and actual no longer holds good." The old rule or saying, *"to be possible* comes before *to be"* ("prius est posse esse quam esse"), may, it seems to me, do very well for things finite, but it can have no applicability to One who is *Ens a se*—Being in and of itself. As such, God is not possibility at all, but the prime metaphysical necessity. Leibniz differs from Weisse when, in dealing with the Anselmic proof, he maintained that if such a Being as God be possible, He exists. For he held that those who would deny this proposition would deny the possibility of Being in and of itself. But if that were so, he says, then all things through another would be impossible, and nothing could exist. That would imply that the necessary Being called God does exist, since we find possible beings in actual existence, but it does not yield an *a priori* argument. But the question of the possible had occupied the mind of Descartes, long before Leibniz, and it is interesting in the present connection to recall

the statement of Descartes,—"But we must make a distinction between *possible* and *necessary* existence, and observe that possible existence is included in the notion or idea of all things of which we conceive clearly or distinctly, but that necessary existence is included in the idea of God alone."

It would be a serious mistake to suppose that Descartes is here making a new distinction. The possible, the necessary, and the impossible, were suggestively dealt with, very much earlier, by that powerful logician Wyclif, whom I have found more interesting in his "Logica" in this respect, than his great opponent, Occam, in his logical "Summa." Wyclif gives the first place to God's existence as absolutely necessary, but speaks of a secondarily necessary as self-necessary, geometrical theorems, for example. "Necessary" means, for Wyclif, "impossible not to be": "Impossible" means "necessary not to be." The meaning of impossibility, for him, answer to those of necessity. Descartes, then, holds that we cannot conceive of God save as existing, and, in his fifth "Meditation," he maintains that, as the idea of a triangle involves its having three angles equal to two right angles, so the idea of God carries with it His existence. It is possible to conceive a mountain or a valley without either of them existing, he says, but it is not possible to conceive any other Being than God to whose essence belongs existence. These are aspects of Descartes' teaching whose tenableness I am not now concerned to discuss; what I am concerned with is, that in the fore-shadowed idea of necessary Being there is already much that would give pause to Weisse's line of argument as to the possibility of God. The only possibility of God that would be left for consideration would be, whether the notion itself of God as the sole necessary Being was one that was intrinsically repugnant, self-contradictory, or impossible, in the character of its Being. If the possibilities

of all things derive from God, as the sole necessary Being, then clearly He Himself cannot be subject to the law of possibility. Aquinas said, "Deus est actualitas totius possibilitatis." But I do not dwell on these aspects, though it did not seem possible wholly to avoid their discussion, because it is the law of general possibility, with which this paper is mainly concerned.

The philosophy of possibility has to contend against many injustices. Pure empiricists will have none of it as a theory, from Hume onwards, although Hume did not deny the possibility of knowledge. The actual is their limit of possibility, its only measure. Bain and Mill are examples. The universe is for such empiricism just what it is, a closed system, impervious to influence from without. Said Lewes, "nothing really exists till it exists, and nothing exists possibly, for possibility is only the uncertainty of our ignorance." Mill talked of "possibilities of sensation, although abstracting all substance and causality in such a manner as to leave said "possibilities" deprived of the conditions of possibility. But the philosophy of possibility is not furthered by certain empiricists telling us that the universe might just as well have been one in which 2 and 2 would have made 5, or the square would have had the form of the ellipse. It is concerned only with possibilities that are real and rational. But what do we get from an idealist like Bradley? When dealing with the principles of logic, he says that "reality in itself is neither necessary, nor possible, nor impossible." For these predicates, he holds, exist only in our reflection. And if our knowledge and reflection were great enough to take in all the facts, "nothing would ever appear possible. The real would seem necessary, the unreal would seem impossible." But, as we are not such impossible beings as his supposition demands, the more relevant task is to discuss possibility as it exists for us. We are only such beings that

we have to accept the law of uniformity of nature, although no final logical reason is possible to us for the huge assumption that nature is uniform. Scientific inquiry resolves itself, it would seem, into the study of possibilities. A scientific hypothesis is but one conception among alternative possibilities. Kepler is said to have made nineteen false hypotheses regarding the form of the planetary orbits, and the theory in which he finally rested, that these orbits are ellipses, was but a possibility or an hypothesis until verified by facts. Uranus first became a possibility to Herschel, and Neptune to Leverrier through gravitation laws, and then they became facts or discoveries. The possible is futurist in character, although there is a sense, of course, in which the Actual may be said to be, before all things possible. But that is a sense with which we are not here concerned, and besides, the question of possibility cannot be raised as to present or actual fact, but should be kept as a question of the future. Possible relations the scientific inquirer seeks to establish, as when, for example, Faraday attempted to discover a possible relation between gravity and electricity. Another example of possibility that had to be considered was, the possibility of expressing the conditions of motion by means of differential equations, while another instance has been the discussion, with reference to modern electrical theories, of the possibility of ultimate mechanical explanations. In the biological sphere, beyond the chromidial unit lay, as an unforeseen possibility, the cell, awaiting discovery. But why enter on examples when the whole progress of science has been strewn with theories of possibilities which have in time been replaced by other and more perfect theories? The field of physical possibility seems of boundless scope, and this is the very inspiration of science. But there are other fields of possibility, such as psychic possibilities, moral possibilities, logical possibilities, metaphysical possibilities,

epistemological possibilities, and so forth. They are not less real after their kind than the physical possibilities. Of the possibilities in every sphere, one may say that they must at least be possible to thought. Anything, the concept of which is self-contradictory, is itself impossible. Kant does not treat the categories, among them that of possibility, with the clearness and exactitude that might be wished, accepting them, as he does, in a merely empiric way. He acknowledges a pure use of the categories to be possible, that is, not self-contradictory, but says that such a use has no kind of objective validity. The categories contain mere possibilities and depend on experience for their validity and confirmation. And experience depends on the categories for its possibility. Thought is, however, no guarantee of reality. Kant failed to appreciate the dynamic aspect of the world, and viewed each category too much as separate and complete in itself. There was lack, therefore, of developmental view of the whole. In an abstract sense, what is thinkable, is possible. This would appear to be the use of the possibility category most accordant with Kant's system. But it is present only in the sense that anything is possible which is conform to the formal condition of experience. His first postulate of empirical thought is, in his own words, that "that which harmonizes with the formal conditions of experience is possible." Kant recognizes possibility of a real or empirical character, which is not without some kind of empirical basis to rest upon. It has been objected that this form of possibility is vague and indeterminate, but that can hardly have much weight with us after all that has been advanced in the foregoing pages. A potency contained in its cause, e. g., is surely a fairly determinate possibility. I think there is something in Cohen's position, that the possible is not the real only in concept; that possibility is a synthetic determination of relation; and that it cannot be drawn off from

reality within the context of experience.[1] How different it may be noted, is the impossible, which remains mere thought—thought vanquished by the real; for in the impossible, thought and reality are mutually opposed. It is just the value of the categories that they render synthetic *à priori* judgments possible, and so advance our knowledge of reality. In such judgments, the categories are the *à priori* elements, conferring on them necessity and universality, and so rendering them scientific. Otherwise, objects might be given to us in experience, but they would not be known.

I have been mainly concerned to show the reality of the possible. If we took it as the merely conceivable, out of all relation to everything else, we should have a merely abstract and empty possibility. But as real possibility, possibility or capacity concretely conceived, it is a highly important category. For this is dynamic possibility, force that is pressing towards expression. For possibility is but a moment in the movement towards existence. Existence is *complementum possibilitatis,* so, at least, Leibniz and Wolff styled it. Wolff was, of course, right enough in presupposing that not everything thought possible is also real. And perhaps his position, that the existence of a thing is a completion of its possibility may, in the view of some, be allowed to stand, in the case of what we have called real possibilities. But it will not stand for possibility in general—not for those abstract possibilities already discussed. For he was mistaken in supposing that everything, which happened to be essentially free of contradiction, has the capability of existing. We cannot assume that, in the case of something abstractly possible, but not actual, its non-existence merely means lack of existence. Exception may be taken to Wolff's position that existence is completion of possibility, even in the case of

[1] H. Cohen, "Kant's Theorie der Erfahrung," p. 234.

real possibilities, for if possibility involves that a thing is already fully determined, it may be contended that it is not susceptible of further determination. What I have already said of Aristotle's positions should be remembered here, but attention must be fixed on what the concept of real possibility involves, such possibility being always determinate. Real possibility belongs to something that realizes itself. But there are differences in the possible here. There is possibility as we see it where some effectuating cause is at work. And there is possibility where, on a teleological view, our own self-conscious purpose is a determinative influence at work for the realization of an end. There may be several possible ways of reaching this end. An egg in virtue of its evolutive property produces a bird, but the egg remains for itself an egg. It is thought which sees in it the possibility which is eventually realized. The possibility of development shows potency thus becoming real. There is this peculiarity, then, in possibility, that the real or existent conditions are complemented by the thought or ideal conditions. This supplementary functioning by thought is very important in the study of possibility. For one may be very skeptical indeed as to possibility being given us through empirical perception; it is thought or reflection upon the experience which finds for us the possible, whether present or future. We cannot be content with the given in perception; the Spirit by means of thought frees itself from the impression. We only reach the concept of real possibility as we think of a continuity, which underlies the changing experience. Logical concepts may stretch out beyond experience, but it is otherwise when we deal with real possibility. For this is just the potency of the real, and possibility may here be limited by dependence on circumstances. The potentiality is merely "an antedated, presupposed, and hypothetical actuality." I have shown real possibility to have a certain

dependence on something empirically given, an actual basis in which part of the possibility-conditions is already realized, and yet it is the very triumph of the category of possibility that it frees us from the dominion of the empirically given. If I think of the world itself as one of unrealized possibilities, set in a universe of many other worlds, also of unrealized possibilities, I am away beyond all experience, I am emancipated from the thraldom of the empirically given. Thus does the category of possibility, with its illimitable horizon, raise me above empirical reality. But it is the insistence of Kant that the use of the categories does not extend beyond the limit of the objects of experience. But the possible is still the thinkable, and the thinkable blends with the intuitional in the formulation of possibilities. Reason plays important part in this realm of posited possibilities, whose realization it demands. Science has continually to reckon with possibilities which it has not yet found in sensible reality. In the more abstract domain of mathematics, there is a whole world of possibilities, with chains of necessary connection. There are necessary possibilities, it must even be said, connected with reason's ideal of reality, and when the idea of the absolute, a possible and necessary idea, has been thought, the limit of the category of possibility has been reached. Away from the sphere of the necessitated, in the region of personal spirit, there is the greatly neglected notion of spontaneity, which yields real possibilities. The possibility of communion with others, in matters of knowledge, belongs to the same personal sphere.

It will be evident that a great deal has been said to prepare the way for the statement that there are degrees of possibility. The possible does not always carry one and the same meaning, a fact which furthers this result. The whole philosophy of probability, with which we are not here concerned, rests upon the power to differentiate be-

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tween different degrees of possibilities. Probability is not only the guide of life; it is the guide of science also. From fantastic and merely conceivable possibilities, which constitute the lowest degree of possibility, we pass up to high degrees of real possibility. That is to say, we pass from the barely possible up to the highly probable. And we have already noticed such differences in kind as logical possibilities and ontological (or objective) possibilities. When it is a question of propositional truths, the possibilities are matters of logical grounds and consequences, not of objective conditions and results. And anything is possible, logically, that is conceivable, that is, so far as it is not self-contradictory. But this, of course, carries no reality coresponding to it. Possibility is another matter when connected with empirical reality. There the possible, whose causes and conditions exist or will come into existence, must be consonant with, or conform to, the laws of Nature. But when, in respect of degrees of possibility, actuality is pronounced to be the maximum of possibility, I doubt whether we should so speak, for in actuality, as it appears to me, possibility is no longer present, but has been already sublated, or, if you prefer, sublimated. We saw that hypotheses were alternative possibilities, not all of which are, in like degree, possibly true. The vast scope for degrees of objective possibility, in the outer world, needs no emphasis. Things may be possible, too, without being known to be so. There is not only our subjective recognition of outer possibilities, but there are the subjective possibilities connected with our own purposes and endeavors, in which also there are degrees of possibility as to their realization.

The great value of the category of possibility, particularly for science, is, I think, its lesson of the need for the open mind in respect of the future. But, of course, this applies also to the philosophical questions, and theories,

which arise out of scientific problems or determinations. Many a scientific hypothesis, and many a philosophical theory, have had to succumb to the tests of ever-advancing thought. The same possibility awaits many of those current today. Evolution by means of natural selection; the turning of thought from mechanical to vitalistic tendencies; the postulation of energy as a quantity constant in amount; the theory of heat; the theory of relativity; the theory, have had to succumb to the tests of ever-advancing tist theory; the theory of value; these, and many more, have possibilities that lie hidden in the future, favorable or otherwise. In the sciences of Nature, possibility amounting to probability is all that we have. But a short time ago, it did not seem possible that any chemical element could change; but radium burst the bonds of seeming impossibility. On the philosophical side, there is the question of open or closed systems. A closed system is one which contains no real possibilities. Every event is to it either actual or necessary. The actual, too, is supposed necessary, and there is nothing possible in such a system but the necessary. New potentialities, of course, there may be, but all is predetermined and fatally certain. The system being closed, there is no purpose: it is non-teleological. There are systems, however, like that of Leibniz, for example, with its pre-established harmony, which, though closed, are not non-teleological, although there is still too much fixedness for the freedom of the moral life. The Hegelian system seeks to mediate between the idea of a finished universe and the freedom of the moral life, but it must, strictly regarded, be taken as, in reality, a closed system. The Hegelian "Logic" expressly says that there must be no talk of possibility or of the possible. We must bring out, says Hegel, the necessity hidden behind every semblance of contingency. In such a system the possibilities are merely apparent, not real and genuine.

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Against all this, one must stand for an open system—a growing or unfinished universe, with room for the free, creative possibilities of the good will. There must be no limit set to the possible increase of ethical and other higher values.

JAMES LINDSAY.

IRVINE, SCOTLAND.

DEWEY'S THEORY OF VALUE

I

IN THE BELIEF that John Dewey's theory of value is not only the hub of his philosophy, but that it also constitutes the chief contribution of Pragmatism to current philosophy, I wish, in this paper, to show what Dewey's theory of value is, to indicate its relations and significance, and to adduce corroborations of the theory from certain unexpected quarters.

No one can hope to understand Dewey's theory of value who does not grasp rather fully the meaning that he attaches to the term "experience." Such an intellectualistic penumbra hangs over the word that some readers never understand that Dewey means other than what they would mean if they used the word in a similar connection. That he does mean something different, and something more, he has emphasized most vigorously in the introduction to his *Essays in Experimental Logic*. It seems that Professor Dewey himself did not for a long while get fully clear the significance of the distinction; and he gives Mr. S. Klyce credit for pointing out to him this and "other indispensable considerations." Upon this point Dewey explains: "Our words divide into *terms* and into names which are not (strictly speaking) terms at all, but which serve to remind us of the vast and vague continuum, select portions of which only are designated by words as terms."[1]

[1] *Essays in Experimental Logic*, p. 8n.

Now the word "experience" and such other words are not terms at all; they are what Mr. Clyce calls "infinity and zero" words. As opposed to such a word as "desk" (which is a term referring to a definite object, on which consciousness is or may be focussed), the word "experience" refers to the fringe or penumbra of the situation in which "desk" or any other term is the focus. As Dewey says: "The word 'experience' is . . . a notation of an inexpressible as that which decides the ultimate status of all which is expressed; inexpressible not because it is so remote and transcendent, but because it is so immediately engrossing and matter of course."[2] Such a word, then, connotes what is before and after and around that which at any given time is denoted. "I shall only point out," says Dewey, "that when the word 'experience' is employed in the text, it means just such an immense and operative world of diverse and interacting elements."[3]

Experience, as this will indicate, is a much broader term than knowledge. Instead, therefore, of putting the question as some philosophers have, i. e., whether there are different ways of *knowing*, we must cease begging the question and ask whether there are not different ways of *experiencing*, of which knowing is only one. We certainly must be content to put the matter in this way, if we are to understand Dewey; for Dewey makes this point in wholly unambiguous language. "Knowing," says he, "is one mode of experiencing, and the primary philosophic demand . . . is to find out *what* sort of an experience knowing is—or, concretely how things are experienced when they are experience *as* known things. . . . To assume that, because from the *standpoint of the knowledge experience* things are what they are known to be, therefore, metaphysically, absolutely, without qualification, everything in

[2] *Essays in Experimental Logic*, p. 10n.
[3] *Essays in Experimental Logic*, p. 7.

its reality (as distinct from its "appearance," or phenomenal occurrence) is what a knower would find it to be, is from the immediatist's standpoint, if not the root of all philosophic evil, at least one of its main roots."[4]

While Dewey has carried the analysis of experience somewhat into detail in the foregoing citation, we may rest, for the purpose of this paper, in his usual dichotomy. If we analyze the various aspects of our experience, we find on the one side what Dewey refers to variously as "appreciation," "realization," "direct, non-cognitive doing-suffering," and on the other side what he refers to as "reflection," "thinking," "cognition," "judgment." Experience, of its own intrinsic nature, falls thus into a dualism—qualitative at least—and the nature and relation of these two parts constitute both ethics and logic. Perhaps nowhere better than in *Democracy and Education* has Dewey plainly set over against each other these two aspects of experience.

"To value means primarily to prize, to esteem; but secondarily it means to apprize, to estimate. It means, that is, the act of cherishing something, holding it dear, and also the act of passing judgment upon the nature and amount of its value as compared with something else. To value in the latter sense is to valuate or evaluate. The distinction coincides with that sometimes made between intrinsic and instrumental values. Intrinsic values are not objects of judgment, they cannot (as intrinsic) be compared, or regarded as greater and less, better or worse. They are invaluable and if a thing is invaluable, it is neither more nor less so than any other invaluable. But occasions present themselves when it is necessary to choose, when we must let one thing go in order to take another. This establishes an order of preference, a greater and less, better and worse. Things judged or passed upon have to be estimated in relation to some third thing, some further

[4] *Influence of Darwin on Philosophy*, p. 229.

end. With respect to that they are means, or instrumental values."[5]

From this quotation as a basis, Dewey's theory of value may, I think, be fairly summarized in the four following theses:

II

1. *Experience, which furnishes the context of all values, is largely non-cognitive.*[6] It is well to emphasize this non-cognitive basis of cognition itself; for upon this plane of experiencing lie most of the contents of our living. Here are included our loves and our hates, our eating and our sleeping, our friendships and our animosities, our illness and our health; here too are the fine arts;[7] here the dumb gladness that welcomes the dawn, the quiet contemplation of the sun's trailing glory at eventide, and the silent watching of the passing night. This is the primal and ever the larger aspect of human life. It is the good-in-itself, from which reflection rises and for whose sake reflection exists as an instrument.

2. *Experience becomes cognitive only when incompatibilities demand more than mere appreciation for their successful resolution.*[8] Dewey is primarily interested in intrinsic values, in the appreciative life described above. Indeed, no living being, thinks he, ever becomes interested in "extrinsic" value until he must in order to save and extend some of the "intrinsic" content of his appreciative life. Then judgment comes into play as an instrument that is justified by resolving the difficulty back into a situa-

[5] *Democracy and Education*, p. 279.

[6] "Experience is primarily an active-passive affair; it is not primarily cognitive." *Essays in Experimental Logic.*

[7] "For Dewey's feeling estimate of the place of art in life, see *Reconstruction in Philosophy*, p. 212. Cf. *Human Nature and Conduct*, p. 159ff.

[8] "Difficulties occasion thinking only when thinking is the imperative or urgent way out. . . ." *Reconstruction in Philosophy*, p. 139. *Cf. Democracy and Education*, p. 280.

tion that permits intrinsic goods again to become possible. If judgment begins building upon itself a hierarchy that forbids the energy of life to dip again into its stream, it meets Nemesis upon its upward way, who robs it of its vaunted glory and leaves it quite inane. "In fact, 'good' is an empty term unless it includes satisfactions experienced."[9]

3. *Such adjustment*[10] *arises from, exists for the sake of, and dips again into, non-cognitive experience.* Thinking becomes thus an instrument, but an indispensable instrument, for the continuous preservation of the values for which it exists.

4. *Moral judgments do not discover value outside experience, but reconstruct and create values within experience.* Let it be carefully borne in mind what has been said about the genesis of judgments of all kinds. Value judgments are not unique, as Dewey thinks, save in that they deal with a content that in direct experience was valuable. Value judgments arise out of a situation made embarrassing by a conflict between two equally valuable parts of experience or out of some other equally unsatisfactory turn of experience; and the judgments are but citations of what seems necessary to make experience once more satisfactory. Judgments are—if one wishes to put it so—discoveries of what, under the circumstances, one ought to do, if he is successfully to resolve the unsatisfactory situation. The judging process is not only the discovering of what is to be done, but it is also the first step of the action itself. It is not a discovery of something already existent (though it has such as its data), because the situation to which the judgment looks as an end that is to resolve the maladjustment, has never yet existed and would not come to exist but for the process of action which is initiated by the judg-

[9] *Democracy and Education*, p. 412.

[10] "It comes after something and out of something, and for the sake of something." *Essays in Experimental Logic*, p. 75. See also *Ibid.*, p. 36.

ment itself.[11] The restored equilibrium is the creation of the active life through the judgment as its tool. "At whatever risk of shock," thinks Dewey, "this doctrine should be exposed in all its nakedness. To judge value is to engage in instituting a determinate value where none is given."[12] The contention that judgments of value are practical includes two points: "one, that the judgment of value is never complete in itself, but always in behalf of determining what is to be done; the other, that judgments of value (as distinct from the direct experience of something as good) imply that value is not anything previously given, but is something to be given by future action, itself conditioned upon (varying with) the judgment."[13] For an ill person to decide that it is well for him to see a doctor is but for him to take the first step in the creation of the good of his then situation, i. e., the seeing of the doctor, a good that but for his judgment would never come to exist at all.

It is well for Dewey to insist upon the practical nature of judgments of value: for if he can establish that fact, he has not only added something to our understanding of such judgments, but he has also in a way as effective as it is indirect warded off from his general theory criticisms from both idealists and realists. From both schools alike have come charges of subjectivism against pragmatism. Those who incline toward subjective idealism have wished to claim Dewey; those who have espoused the cause of objective idealism have joined with the realists in saying that Dewey is a subjectivist. Only in so far as Dewey's general theory of knowledge is involved in his theory of value am I interested here in discussing his views on epistemology. Dewey has elsewhere attempted to show that those who, with Bertrand Russell, state the existence of

[11] *Essays in Experimental Logic*, pp. 358-359.
[12] *Ibid.*, p. 368.
[13] *Ibid.*, p. 361.

the world as a logical problem are but unconsciously chasing their own shadows; for the very words in which they are forced to state the problem show that they have already answered the question in the affirmative before the question is put.[14] But as regards the epistemological gulf in the study of value, if it were true that there is such a thing as the objective world and such a thing as the subjective world, and the integrity of the subjective were determined by its "correspondence" with the objective, then it were a most serious charge against Dewey to say that he is a subjectivist. For in the field of value, it means that he proclaims what could be hardly less serious than moral solipsism. To both the idealist and the realist who feel that they must have the objectivity of moral judgments before their judgments can command respect, Dewey seems, if not himself corrupt, at least a corrupter of youthful America.[15]

How, then, does Dewey escape the seriously meant charge of denying the objectivity of value? He escapes by pleading a change of venue, by denying the jurisdiction of the court. "I can but think," says he, "that much of the recent discussion of the objectivity of value and of value-judgments rests upon a false psychological theory. It rests upon giving certain terms meanings that flow from an introspective psychology which accepts a realm of purely private states of consciousness. . . . To refer value to choice or desire, for example, is in that case to say that value is subjectively conditioned. Quite otherwise, if we have steered clear from such a psychology."[16]

Once granted that there is another way of approaching the problem than the subjective-objective route; i. e., once granted that not all experience involves the knowing relation, one will have little difficulty in seeing that Dewey's claim that value-judgments are practical, is an effective

[14] *Logic*, p. 281ff.
[15] W. H. Sheldon, *The Journal of Philosophy*, 18:309-20.
[16] *Essays in Exp. Logic*, p. 364. See also *Democracy and Education*, p. 195.

refutation to either the claim that they are subjective or the claim that they are objective. Even though one himself does not accept all that Dewey puts into his conclusions, he must then admit, I think, that Dewey is at least consistent with himself. This robbing of the subjective-objective problem of its meaning is by no means the least thing for which students of value have to thank Dewey, as I see it. So long as we did not see that value-judgments arise out of a specific non-cognitive situation in need of adjustment, that it is the first step in an action that seeks to resolve the difficulty, and that it dips then to a non-reflective level where intrinsic goods are again possible—so long did we wander from the meaning of concrete experiences and lose ourselves in the attempt to discover the good, *überhaupt*.

This has been the besetting sin of philosophers beginning with and including Plato.[17] But whether they have been on the search for *"the* end, the *summum bonum*, the final goal" or for inflexible standards with which to sound conduct for "eternal values," they have uniformly returned empty-handed. The sufficient reason why they have failed, says Dewey, is that "in the abstract or at large, there is no such thing as degrees or order of value." "It is reasonable to believe," says he in another place, "that what holds moral knowledge back is above all, the conception that there are standards of good given to knowledge apart from the work of reflection in construction of methods of action."[18] The reason for this faith on Dewey's part he elsewhere indicates in these words: "Physical knowledge did not as matter of fact advance till the dogma of models or forms as standards of knowledge had been ousted. Yet we hold tenaciously to a like doctrine in morals for fear of moral chaos."[19] Just as the abolition of this point of view

[17] *Influence of Darwin on Philosophy*, p. 50.
[18] *Logic*, p. 382.
[19] *Ibid.*, p. 381.

in the natural sciences has led to such phenomenal widening of the boundaries of our knowledge, even "in like fashion," thinks Dewey, "we may anticipate that the abolition of *the* final goal and *the* single motive power and *the* separate and infallible faculty in morals, will quicken inquiry into the diversity of specific goods of experience, fix attention upon their conditions, and bring to light values now dim and obscure."[20] Renouncing, then, once for all "the diversion of intelligence from discrimination of plural and concrete goods . . . which has done more than brute love of power to establish inequality and injustice among men,"[21] and leaving henceforth to "poetry and to art, the task (so inartistically performed by philosophy since Plato) of gathering together and rounding out, into one abiding picture, the separate and special goods of life,"[22] we shall "converge all the instrumentalities of the social arts, of law, education, economics, and political science upon the construction of intelligent methods of improving the common lot."[23]

III

It is this ringing call for man to live in his own world and this justification for his so doing, that constitutes the essence of Dewey's philosophy. While others have cried "lo, here; lo, there!" Dewey has continually insisted that the kingdom of good is within human experience. It is difficult to see why this course should need to be emphasized. For other-wordliness is supposed to have passed with many other anemic beauties of the mediaeval world. And this is all that Dewey is really saying in his theory of value: values are immanent in human experience. Take them for what they are: if they are many, so much the better; if

[20] *Darwin*, p. 70.
[21] *Ibid.*, p. 75.
[22] *Ibid.*, p. 71.
[23] *Ibid.*, p. 69.

they are found within classes that aforetime were vulgar, judge the classes by the values, rather than the values by the classes. When Dewey insists that values are *here,* he also indicates the very important social doctrine that they are *everywhere* here. No class has a monopoly upon them; if so, the chief task of man is to see that monopoly ended. To make values common to all men, to deepen them, and to guarantee them—this is the threefold problem common to philosophy, to science, to government. This is the Problem of Man. The first step in the solution of the problem is the whole-hearted recognition that values are immanent in human experience, rather than secluded in some transcendental or conceptual realm accessible to common men only through priestly or philosophic or governmental intermediaries. It is this recognition that philosophers have often failed consciously to make. I say "consciously," for since all the values there are, *are* really in human experience, it would not be marvelous if philosophers were found, when off guard, to speak more wisely than when in the cold ecstasy of philosophic sophistication. With this hypothesis let us ramble a bit in the out-of-way preserves of the two modern system-builders.

Of all moderns who have found human experience most lacking, whether taken as a whole or by parts, F. H. Bradley perhaps comes first. His *Appearance and Reality* succeeds so fatally well as practically to mean for man, as Schiller facetiously suggested, the *disappearance* of reality. Begin where he will in the evaluation of human experience, Bradley can find nothing that is not shot through and through with inadequacies, contradictions, infinitude. "Surely," he would say, "nothing here can be thought to be finally valuable." It is only in the Absolute that value can be found (though he even quibbles at predicating good of the Absolute).[24] Here, it would seem, is a complete

[24] *Appearance and Reality,* p. 411.

antithesis to Dewey's theory of value; for Dewey finds all values in human experience, whereas Bradley finds none possible in it. If human experience has any values at all, they can appear as valuable only in the Absolute, never in themselves. Here we see a traditionally recurring effort to belittle man's world carried through to its logical completion. Man's world remains good only for its bads.

But the Absolute, in relation to which alone can anything else possess value, gets its value from its reconciling nature. The Absolute seems the complete embodiment of the seventh beatitude: in it all the contradictions of a bedeviled world are set right. In it all questions cease from troubling and all problems are at rest. But how is this consummation (so devoutly to be wished!) attained? It is attained by the abolition of thought (relationally infected thought!); for the Absolute is not only trans-temporal, trans-spatial, but trans-rational as well.[25] If solution of contradictions is the great end that the Absolute attains, then absence of reflection is the great means through which the great end is reached.

At this stage, the question will inevitably arise: If the solution of contradictions is the end to be sought, and the means to its solution is the absence of thought, then why go to such length to attain what is already present at the outset? For unless the term "thought" be used in such a double sense as really to destroy its meaning, thought certainly does have its *genesis* in just the sort of situation that Bradley thinks it attains only at the end. The appreciative realm in which we daily live, move, and have our being is, as such, non-reflective; and being non-rational, it has neither contradictions nor problems. These do arise, but their birth means its death. We do not value and evaluate the same object at the same time. Whether one recognizes it or not, it is the deep concern with which

[25] *Appearance and Reality*, p. 172.

he views the interruption of his a-logical experiencing by a hostile environment, a concern growing out of the unspeakably, the un-rationally profound significance this mode of experiencing has for his life,—it is this concern, I say, that leads to such persistent efforts, as Bradley's, to think out the contradictions and restore the values. In his description of the Absolute, Bradley has described with vivid accuracy a genuine part, the most meaningful part, of *human* experience. In a paradoxical epigram, half-playfully placed in the preface to his *Appearance and Reality* (an epigram apparently evolved in a moment of detached musing), Bradley has done more than in the entire book that follows accurately to analyze human experience and precisely to indicate the relations between its parts. Says he: *"To love unsatisfied the world is a mystery, a mystery which love* satisfied seems to comprehend."[26] (Italics mine.) Not even the incorrigibly intellectualistic sentence[27] with which he closes the excerpt can obscure the genuine significance that Bradley himself finds in immediate experience, such as is represented by "love." Indeed, Bradley's epigram indicates in a sentence Dewey's own theory of value. Bradley's naïve insight at the beginning states, it seems to me, quite as intelligibly as does his much belabored dialectic that follows for six hundred pages, the nature of thought and reality and their relations to each other. Since Bradley graciously leaves to the reader "how seriously"[28] he shall take that epigram, I for one have long preferred to take it more seriously than I do the remainder of *Appearance and Reality*.

Another corroboration of Dewey's analysis and emphasis that comes from quite as unexpected a source and comes, even as in Bradley's case, from a moment of relaxation

[26] *Appearance and Reality*, p. xv.

[27] "The latter is wrong only because it cannot be content without thinking itself right."

[28] *Appearance and Reality*, p. xv.

rather than from a serious intellectual endeavor, is found in a recent note of Bernard Bosanquet's.[29] Bosanquet, who through labored volumes loses human experience and values in a conceptual Absolute at their close, Bosanquet, whose "inmost aspiration" [30] was, and is, to be able to say to the "critics of Absolutism": "Mark now, how a plain tale shall put you down,"—even Bosanquet has here told so plain a tale as to justify the feeling that the Absolute may be brought down as effectively by its friend as by its enemies. Bosanquet, surveying the *Studio,* feels an obvious impatience with those who (following his own footsteps) would push values out of human experience. "In a world of supreme values, wholly beyond doubt," he found himself saying, "What is the use of talking? Why do we not look?" There before him, the great apostle of the Absolute, were some pictures, "any one of them fit to bring heaven into our time and place." In that moment of insight, he confesses, "that the *inexhaustibleness* of values, of *human experience,* is *altogether beyond the need of reasoning,*" and the thought left him, as he further confesses, "a little indifferent to the *precise remoter* inferences which we may draw from it, and a little impatient of any discussion which implies that we are not *constantly* in presence of *supreme realities* and *immeasurable values.* (All italics mine.) This "Undesigned Coincidence," as Bosanquet entitles his brief note, furnishes further corroboration of the theory of value enunciated in the earlier pages of this paper. Undesigned these corroborations are, but perhaps of greater weight because they *are* designed; for they indicate an *unbiased* recognition of just such a state of reality as Dewey's theory of value has so logically elaborated.

[29] *The Philosophical Review,* 30:216.

[30] *The Principle of Individuality and Value,* p. vi.

IV

If one must have in every philosophy a *metaphysics,* I wonder if we might not call Dewey's theory of value, his metaphysics? This is suggested to me by the discovery that Dewey is treating in his theory of immediate values what those who have monopolized the term "metaphysics" treat under that head. The Absolute is the metaphysical object, *par excellence;* and Dewey also has his doctrine of the absolute or of absolutes. Every act of thinking finds its specific absolute in the restored immediate experience to which the act is instrumental. Immediate experience *is* absolute in the sense that it is the *end* of the problematic situation, and as such is consequently "invaluable."[31] Immediate experience enjoys the same surcease from the exigencies of thinking as does Bradley's Absolute. It seems to me that all the really distinctive and valuable attributes of the historical Absolute is preserved in Dewey's absolute(s), for the doctrine of the Absolute (absolutists to the contrary notwithstanding) has ever been an effort to guarantee emotional satisfaction through the procedure of rationalization. Dewey guarantees it, not by rationalizing, but by recognizing it for what it is and setting about with scientific foresight to *make* it permanent. In short, Dewey has succeeded in showing how within human experience itself, thought, instead of breeding vast contradictions through its relational nature, is, in the ascent of man, the instrument evolved for the resolution of the difficulties that arise on a lower level. Thought, precisely because it is relational, is the one means through which the absolute(s), which we so much prize, may retain their absoluteness; i. e., may remain unproblematical.

[31] See *Reconstruction in Philosophy,* p. 175.

V

I wish now to return briefly to Dewey's own theory in order to consider what seems to me the most significant objection made to it, an objection raised curiously enough not more by critics than by Dewey himself. Dewey's theory of value, as we have seen, calls for a dichotomy within experience; i. e., appreciation and reflection. But do these actually exist, separated from each other, as the discussion seems to imply? However appreciative it may be, is not all experience judgmental, *implicitly* at least? It is too easy to beg the question with the use of the term "implicit." That all experience, however purely appreciative, may (and does) become judgmental on occasion, Dewey both admits and affirms. Moreover, he explains what is "the occasion." But if the critics should affirm that all experience is *actually* judgmental, they would be doing no more than Dewey himself seems more than once to admit.[32] With this admission one might still maintain that these two aspects—judgmental and appreciative—predominate in different situations, as Dewey himself affirms. But once admit that all experiences are all the time actually infected with more or less of the judgmental aspect, it seems to me that the distinction loses much of its significance. It may still be justified as an explanatory device, but it can no longer be sharply descriptive of actual experiences. Believing that the division is not only explanatory but accurately descriptive of human experience as well, I do not feel that Dewey need make the admission (if the passages quoted below and other similar ones constitute an admission.)[33]

[32] "No experience having a meaning is possible without some element of thought. But we may contrast two types of experience, according to the proportion of reflection found in them." (*Democracy and Education*, p. 169.)

[33] "That something of the cognitive . . . enters in as a catalyzer . . . in even the most aesthetic experiences, seems to be altogether probable." (*Logic*, p. 394.)

The fact that one is always (during waking hours) thinking, leads too easily to the conclusion that all our experiences are more or less thought experiences. True, of course, that all our experiences are thought experiences while they are being thought about. But the question really is: Are all our experiences all the time being thought about? It seems clear to me that they are not, even to the degree that Dewey thinks probable. Thinking is never of things in general, never *überhaupt*. To the contrary, thinking is always quite utterly specific, as Dewey has repeatedly emphasized. Ignoring the apparent facts of sleep (in which one may afterwards distinguish satisfactory or unsatisfactory rest, though he remember not so much as a dream during the sleeping period), I should say that, since thought is genuinely specific, one is always *experiencing* more than he is thinking about; and this "more" (though it may at any time through developing disharmonies become the object of judgment) is for the time being experienced *only* appreciatively.[34] To use Dewey's illustration, I may *value* my meal while *evaluating* the argument of my friend; or I may *evaluate* grit in my bread while *valuing* the presence of my friend. It seems to me that the two may not only be thought of as separate, but that they are actually quite separate; i. e., that Dewey's oft-repeated dichotomy does constitute a social and ethical metaphysics.

T. V. SMITH.

THE UNIVERSITY OF CHICAGO.

[34] It is, I think, because he does not clearly recognize this that Mr. Delton Thomas Howard in *Dewey's Logical Theory* (Cornell Studies in Philosophy, No. 11), is betrayed first into admitting that "it is doubtless true that men think only occasionally and with some reluctance" (p. 124), later into the declaration that "there is nothing in evidence to show that thinking is a special kind of activity, which operates now and then" (p. 127), and finally into the distinction that "the moment of *real*, earnest thinking is at the high tide of life, when all the powers are awake and operating" (p. 132).

HISTORY AND PHILOSOPHY

HAS HISTORY any data or method to contribute to philosophy comparable in importance to the data and method of the physical sciences? I propose to maintain that it has; in spite of the undoubted importance of the physical sciences to philosophy. For the purpose of the argument, philosophy or metaphysics may be taken to mean *the study of the nature and number of what is real;* and this may be held to include the philosophy of Realists and modern Idealists, of Mr. Russell, Mr. Bradley, M. Bergson and Signore Croce. By a mistake of historical interpretation it is frequently supposed that, as physical science advances, so metaphysics recedes; and it is often said that what is now the subject-matter of astronomy or biology or psychology was once subject matter for philosophy. Probably, however, it would be truer to say that when any section of reality or any class of reals comes to be studied in segregation from other sections, when the relation of its elements or its peculiar type of reals comes to be thought of in a special way, then what before was studied simply as real is studied in science as one kind of real. The same "reals" or the same section of reality still continues to be subject-matter for philosophy, as it was before, because it is real: stars, for example, do not cease to be data for philosophy because the laws of gravitation have been discovered. The sphere of philosophy has not shrunk: indeed, in some cases, the new science may in-

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crease the data or improve the method of philosophy. New types of reals may come into sight, by the closer inspection in a new science of a separate section of reals. New ways of arriving at truth, especially in some restricted field, may be found in the detailed work of the physical scientist. Aristotle may have been wrong as a metaphysician in his classification of the heavenly bodies as a unique type of reality: but the modern metaphysician as well as Aristotle must have some place in his universe of discourse for the stars. In the same way recent advance in psychology has not deprived the metaphysician of the right or the duty of placing "mind" among the reals of which he must take account, or of showing that there is no ultimate real referred to in the traditional use of the word "mind." Thus the data of philosophy or metaphysics are actually increased by the advance of special sciences; and a new method, which is found to be applicable in a restricted field, may be useful also in the wider field of the study of the real as such. The place of philosophy is not less, but more, important as new data and new methods come to be known.

From these preliminary considerations it will appear in what sense philosophy may be conceived to gain from physical science. But it is essential, further, to distinguish more clearly what is here meant by history. For the purpose of the argument, history may be taken as the name for all those classes of study which divide the field of *knowledge of sections* of realities with the physical sciences. Thus it includes the study of events in the record of human society or individual human life, the study of literature and the non-philological study of language, the study of music and of art generally,—all that field of knowledge which is sometimes called "the humanities," as contrasted with "science." Clearly this distinction is arbitrary. There are (i) some subjects of study which are on the border line, as for example psychology or geography in

its social aspects; and (ii) in "history" there are elements of science, as appears from the use of statistics. It would be difficult, with the use of the terms here suggested, to say whether the theory of evolution is history or science. Nevertheless, the distinction is adequate for its use here; for there is a sufficient exactness in the distinction to allow of showing what the study of art and human affairs may have to add to the data of metaphysics. The kind of realities studied in history are sufficiently distinct from those studied in physical science if we say that they are "mental" realities, but this does not involve that there is no science of mind. The methods of history, as distinct from science, are sufficiently obvious, particularly in regard to that mental activity usually called appreciation. One further preliminary statement is needed. The form of mathematics which is logic, or the logic which is the basis of all thinking, is not to be identified with the physical sciences. No one denies that mathematics in this, the Platonic, sense is fundamental. It contributes data and method to philosophy; but its contribution should not accrue to the credit of science *as opposed* to history; for history, no less than science, must be based upon, and must use, the laws of number, identity, difference and the rest. It is true that, in what is commonly meant by history, logic does not seem to be, even in the vaguest sense, mathematical; but that is in part a defect of historical practice, not a necessary characteristic of history in its truest sense. In any case, no knowledge at all is possible except in dependence upon the processes, and in reference to the realities referred to, in mathematics in this widest sense of the word. We are not, therefore, comparing the data and method of mathematics in this sense with the data of history.

History adds to the data of philosophy (*a*) mnemic causation, in the language of Mr. Russell's *Analysis of Mind*. This is characteristic, not only of individual expe-

rience in sensation, but also of the racial or group experience which is too much neglected by skilled psychologists. Matter, indeed, in the study of physical sciences seems to preserve its past experiences. Metal struck once is in a new state to receive a second blow; but clearly mind includes, or is affected by, or represents, past experience in some way differently from this. The manner in which mind so becomes the past in the present is set out in history. This, of course, does not assume that there is "a" mind, or Mind in the traditional sense; for it is equally true of history, if mind is a perspective or a "section" of material things. That kind of reality, which is referred to when we speak, not of "the real pen," but of "the pen in thought or mind" (what used to be called "the idea of the pen"), is studied in history as part of a process in time. And of that kind of reality one characteristic is mnemic causation, which is of very great importance to philosophy. Indeed anyone, who, with a very full knowledge of chemistry or physiology, set out to reach a philosophy *without* a knowledge of mnemic causation, would be likely to misrepresent "the nature and number of what is real"; nor would infinite progress with his physical science supply the omission, although it might suggest that there was an omission. Causation in the world of chemistry or physics would not indicate the peculiarity of causation in the world of mind or thought. There is, of course, no peculiarly superior status to be given to mnemic causation; but its difference from other types of causation is important for philosophy. The characteristics of this type of causation as presented in history will be described later.

Secondly, (*b*) history adds to the data of philosophy uniqueness or identity of events and of "persons." In history an event or a person is not simply nor chiefly an example of a law or a specimen of a class, as they tend to be regarded in science. Of course, history does include some

study of "laws" in the sequence of events, and many speak as though we could "learn" from history by reference to similarities between our own situation and some other situation in the past. This implies the scientific element in history, but it does not include that other element in history which is referred to in the saying that "history *never* repeats itself." The uniqueness of each moment, or point-instant (in Professor Alexander's language) in the historic series is a fundamental fact. But philosophies much dominated by scientific conceptions seem to imply that we could explain the unique moment or individual by reference to a law or an all-absorbing force or reality within which the uniqueness disappears, or of which the unique may be conceived to consist. Bergson, for example, in his *élan* seems to explain away the uniqueness of the moments in a process; but it is bad philosophy to treat as "explained" what has been omitted, nor is it possible to suppose that uniqueness or individuality is not "objective," but only the creation of a spatializing mind. It is "given" just as obviously and irrefutably as any process; and this *datum* is presented in the study of history. Science also, if it is in the widest sense mathematical, presents the unique and the individual or particular; for in any section of space the points are each unique and particular; and in sciences implying evolution there are point-instants in the process described as development. But in history, especially of human or mind process, the instant in the process of time is in a new aspect seen to be unique and particular. What is this new aspect? It can best be understood by reference to the misleading implications of the common idea of progress. The majority think that one generation exists or works for the next, that the future is in some sense the justification or the explanation of the present and the past; but this is clearly a mistake. Whatever the results of my action upon the fortunes of future generations, I exist, so

to say, in my own right and not for the sake of what is to come. Each event, each instant in historical process is what it is, independently of its relation to other members of the series. The excellence of a state of mind is not entirely to be tested by reference to its results on the future, and this is the kind of excellence to which George Meredith refers when he speaks of a kind of reality:

> "Whose fleetingness is bigger in the ghost
> Than time with all his host."

The importance of time or of "motus," as the scholastics called it, or of *durée,* is well recognized by M. Bergson; but he seems to have misinterpreted the character of time in omitting to give an important and permanent place in his interpretation to the uniqueness of the event or the point-instant. As it has been seen, the phraseology of Professor Alexander in his *Time, Space and Deity* has been used here, and a further conception from the same source may emphasize the importance of history in this matter. He speaks of time as the "mind" of space or of a similarity between time-space and mind-body; but this "mind" aspect of time is more clearly to be seen in history than in science.

Thirdly (*c*) history gives the characteristics of mnemic causality and mind or thought in general. For example, under the general term, history, we have included the study of arts. Arts are peculiar to humanity. The processes and products of the arts are parts of the real world and are important to philosophy. To neglect them in a systematic metaphysics is to omit facts, and they cannot be rendered in the terms of science. A product of art, a painting, a melody or a novel or poem, is not, as Aristotle thought, an imitation, and even the wildest reinterpretation of his terms will not prove him to be on the correct lines for the placing of art-products in the world of realities. That element in the painting which is not the amount of the paint

and canvas nor the spatial relation of the parts, that element which makes us say of it that it is beautiful or not beautiful, is not secondary; nor is it reflection of anything else. It is "in" the painting. It is a unique creation. To know how it came there, who was the painter, and other facts "about" it, is not to know *it;* but *it* is the subject for knowledge for one section of history as the study of "the humanities." Now we have no reason to suppose that any other reality but "man" can produce or appreciate art. The perceptions by animals of sounds or of likenesses in pictorial art is no proof that they perceive or appreciate that particular "it" to which a person who knows what a good painting is refers when he says it is beautiful.

In this section of the data provided by history the "group" characteristics of thought or conation or mental activity in general should be included. All art is social. Indeed all mental activity is social. This may seem to be very obvious; but psychologists, logicians, and even philosophers tend to forget it. For example, in logic it is often said that language is an instrument of thought; but clearly its main characteristic is to be an instrument, not of thought, but of *communication,* and "of thought" only because communication is essential to thought. Indeed, all the so-called laws of thought and rules for deduction and induction are spoken of as if "a" mind were active in a non-mental world; but we know of no such mind. All we know is minds in the plural, always in relation to one another. Conation and, still more, "feeling" are social. They are mental activities only as of *many minds;* and of this fact philosophy must take account. But its importance as a fact in the real world is nowhere more clearly to be seen than in history. Finally, truth, goodness and beauty, all of which are in some sense realities, are social; and cannot be understood except in reference to more than one mind, as Professor Alexander has shown in *Space, Time*

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and Deity. But the operation of these realities and their connection with non-mental realities are to be seen in history.

This fact, the social character of mental activity, is not only important as a *datum,* but also because it indicates a method in philosophy. The traditional logic as well as the traditional psychology is "atomistic" or individualistic. The judgment is treated as fundamentally an act of "a" mind in a non-mental *milieu;* but clearly no mind exists or acts in that environment or atmosphere. Even the proposition, which Mr. W. E. Johnston, in his new *Logic,* has shown to be an "objective" fact distinct from the judgment, can hardly be understood without reference to the plurality of minds. For example, it is probable that the chief distinction between truth and falsehood, or reality and illusion, arises from the fact that the "real," as an object of thinking or mental activity, is a perspective for *more than one mind.* Illusion is obviously what is *not* for more than one, except in cases where one dominates or expunges the perceptiveness of the "others" in the experience. The interaction of minds is clearly of the first importance to epistemology and philosophical method; and this interaction is shown in history.

Against the arguments used above an objection may be raised as follows: It may be said that great advances in philosophy have followed the discoveries of physical science, but no noticeable effect upon philosophy can be traced to advances in historical research or humanistic studies. To this objection there are at least two replies: First, the actual practice of historians must not be identified with the characteristics of history, and secondly, the experience of the recent past is not typical of the whole of experience, for in the Renaissance and in the eighteenth century history was useful to philosophy. As a preliminary, it must be repeated that advances in philosophy due

to "mathematics" in its widest sense are, of course, incomparably greater than those due to other sources; but, as was said above, what is due to mathematics in this sense must not accrue to the credit of physical science as contrasted with history. As for the practice of historians, it is true that historians are commonly untrained in philosophy or in ethical theory. They accept the social ideas and standards of their grandmothers. They pass or imply ethical judgments without even noticing that they are applying an uncriticized traditional criterion. Nevertheless, history is in essence a sphere for the comparison of ethical standards and for the examination of the character of mind in society. In the Renaissance the humanists, and in the eighteenth century Leibniz and Hume were as much historians as "scientists," in the narrow sense of that word. The data supplied by Hegel, although misinterpreted by him, may fairly be regarded as historical. Finally, one of the defects of contemporary philosophy, especially in regard to the nature of mental activity, is due to the too great dependence on the physical sciences and the too little attention to the data and methods derived from history. Clearly there must be an advance in history itself before it can contribute much to philosophy. The use of records is at present crude, the ethical judgments of historians are commonly primitive, and the criticism of art is "childish"; but in a more advanced stage of "the humanities" philosophy would gain much in data and method. Even as it now stands history has much to offer.

C. DELISLE BURNS.

LONDON UNIVERSITY, ENGLAND.

THE RELATION OF SPACE AND GEOMETRY TO EXPERIENCE

VII. Conflicting Measurements

IN OUR last lecture we defined vectors—i. e., directed distances—in terms of such notions as we had already taken as primitive in the definition of parallelism, and of no other notions. We then defined in terms of these notions and that of some particular spatial region alone, which we suppose to be spherical, in all future applications of this definition, the set of all vectors-in-the-sphere: that is, the class of all directed distances inside our sphere. We defined, again introducing no new notions whatever into our definition, the magnitude of each vector inside a given sphere in terms of the diameter of the sphere as a unit, or as we put it, the *index* of a vector-in-the-sphere. This definition of the index of a vector was so framed as to give perfectly unambiguous indices of the vectors in any region whatever, no matter how it might be shaped, in terms of that region, without involving, for example, that this region should have anything at all analogous with a diameter, but it was also so framed that, in an ordinary Euclidean space, if the "sphere of measurement" is really spherical in shape, the system of measurement of vectors-in-itself defined by it will agree completely with the system of measurement that is characteristic of ordinary Euclidean space.

Our purpose in this lecture is to obtain from the theory of measurement developed in the last lecture a general system of measurement by which we can determine the distance between any two points in space. In our last lecture, we left this problem still unsolved in two distinct ways. In the first place, the theory of measurement which we developed in the last lecture only told us how to measure the distance between two points both of which lie within our sphere of measurement. This leaves open the problem how we are to measure distances between two points of which one or both lie outside our standard sphere. In the second place, we have not discussed in any manner whatever the problem, how we are to determine what our standard sphere of measurement is to be, and how it is to be discriminated from other sets of generalized points. These two questions and the further problems to which they give rise form the subject-matter of this and a large part of the following lecture.

As we have just said, our first task is to find a method of extending the system of measurement which we have developed for all distances inside a given sphere to all distances in space, so that we may be able to compare any distance in space with the diameter of our standard sphere. You will remember that we defined a certain vector throughout space as the extension of a certain vector-in-a-sphere R, when and only when in all the cases where two points that are separated by R they are separated by the vector throughout space in question. It is clear that it is only natural to assign to a given vector throughout space that is the extension of R the index of R as an expression of the magnitude of the vector throughout space. Of course, we have said nothing which makes it a consequence of the definition of the extension of a vector-in-a-region that a vector in space cannot be the extension of vectors-in-a-given-sphere having distinct indices,

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though this cannot happen if our fundamental notions live up to their names. When this happens, we shall say that the vector in space in question possesses *both* of these indices, at least for the present. Since by this attribution of an index to the extension of a vector in our standard sphere we have found a way of measuring distances between points in any part of space, it might seem that we have obtained a satisfactory definition of a distance, which will apply to any distances whatever. A little reflection, however, will convince one that this is not the case. There are certain vectors which are not the extensions of any vectors in our standard sphere, in general. If our standard sphere is one inch in radius, it is obvious that no distance of two inches can be the extension of any vector inside our standard sphere, for there can be no two points both inside our standard sphere separated by a vector two inches long. The question is therefore before us, how are we to measure in terms of the diameter of our standard sphere distances larger than this diameter? This problem is a particular case of the more general one as to how we measure any magnitude with a scale smaller than the magnitude itself. We have that problem on hand, for example, when we wish to measure a yard and a half of cloth with a footrule. We solve this problem, as a matter of practice, as follows: we first apply the footrule to one end of the piece of cloth, and make a step of one foot. We then start from the end of the piece already measured and make another step of one foot. We find that after making in this manner four distinct successive steps, each a foot in length, provided that all these steps have been taken in a straight line pointing directly towards the further end of the piece of cloth, if we now take a step of only half a foot in length, we shall precisely reach the end of the strip of cloth. That is, if we take four successive steps of the whole length of the footrule, and so dispose

them that we are led by them as near as possible to the end of the strip of cloth, we shall have to take a further step of the length of half a foot, measured by the rule, to reach the end of the strip of cloth. It is not essential, however, that our first four steps should each be a foot in length: we might first have taken a step seven inches in length, measured by the footrule, then one eight inches in length, then one eleven inches in length, then one ten inches in length, then one six inches in length, then one five inches in length, and finally, another step seven inches in length. That is, if we cover the space from one end of the strip of cloth to the other in a finite number of steps which are measurable by our footrule and which are so disposed that the sum of the lengths of these measurable steps is as small as possible, we call this sum of all the lengths of these steps the total length of the strip of cloth.

Let us now return to the problem of the measurement of distances which are too large to fit into our sphere of measurement. Suppose that the distance between the point A and the point B is of this kind. Then it will be of course impossible to go from A to B by a single step which belongs to the extension of some vector situated inside our standard sphere, but it may be possible to make the transition from A to B by the intervention of a finite sequence of successive steps each of which is an instance of the extension of some vector in our standard sphere. Let these steps be instances of the vectors which form the extensions of the vectors-in-our-sphere S, S′, S″, ..., $S^{(n)}$, and let the index of $S^{(k)}$ be $i^{(k)}$, where $S^{(k)}$ stands for the kth step in the chain connecting A and B. Let the sum $i+i'+i''+\ldots.+i^{(n)}$ be called I. I represents, then the total length of the chain of linear segments representing the steps by which the transition from A to B is made. The length I evidently depends on the particular chain which we select to connect A and B, and on what particular

vector-in-our-sphere we regard as furnishing each member of our chain as its extension, when a member of our chain represents the extension of two distinct vectors-in-our-sphere. In ordinary geometry, there is a certain minimum length which chains connecting A and B actually possess—that is, I has a certain minimum value, and there are actually chains of a finite number of vectors which are extensions of vectors in our standard sphere which have this value of I as the sum of the indices of their members. Such chains stretch in a straight line from A to B: they exist, for every distance between two points which is in magnitude less than the diameter of our standard sphere may easily be shown to be an instance of the extension of some vector-in-our-sphere, and Archimedes' axiom holds in ordinary geometry—that is, since when any two distances l and m be given, there is some integer k such that the distance kl is greater than m, so that we may get anywhere by a finite number of steps as small as we please. If, however, the collection of generalized points which we select as spherical should turn out, after all, not to be spherical, or if our initial relation of apparent intersection among convex solids should belie its name, we have no proof at hand that there exists a chain connecting A and B whose length is actually the shortest that such a chain can have: there may be chains, for example, in which I may be made to assume any value you please greater than two, while there may be no chain for which I assumes precisely the value "two." It would seem highly unnatural to say that in this situation, which, as we have seen, can never occur in ordinary geometry, A is at no distance from B. We wish, therefore, to obtain a definition of the distance between A and B which will be the least possible value of I when such a value exists, and which will be, to put it crudely, sufficiently like that number to be called naturally the distance from A to B when there is no single value of

I which is smaller than all its other values. The quantity which we thus define as the distance between A and B must further be such as always to exist when A and B are ordinary generalized points. Furthermore, it must not be such as to make our system of measurement trivial too often, by causing too many distances equal to zero, or in some similar manner.

In our last lecture, we took notice of the fact that if we are given any set S of positive real numbers, and if all the members of S are less than some given positive number *n*, there is some single positive real number which we may call *x*, which is at least as great as any member of S, but which is also such that if *e* be any positive real number, however small it may be, there is some member of S greater than *n*—*e*. x is called the upper limit or maximum of S. Whether S be made up of numbers all of which are smaller than some fixed real positive number or not, it may be shown in a similar manner that S also determines a single positive real number *v*—which may be zero—which is smaller than any member of S, but which is such that if *e* be a positive real number as small as you please, *y*+*e* is greater than some member of S. This number *y*, which is uniquely determined by the class S is called the *lower limit or minimum of* S. The existence of the lower limit or minimum of any class of positive or zero real numbers may be proved as follows: let S be the class in question, and let S′ be the class of all positive zero real numbers smaller than any member of S. S or S′ may or may not contain any terms. Let us suppose that S is not the null-class, the class with no members, and that it actually contains some terms. In this case, S′ may or may not contain some members. If S′ contains no member, it is obvious that *e*, however small it may be, is greater than some member of S: that is, 0+*e* is greater than some member of S. This is true because, by hypothesis, there is no

positive or zero number smaller than every member of S, as such a number would belong to S′, which we suppose without members. Since S is made up entirely of positive or zero real numbers, it can contain no member less than zero. Consequently o is the lower limit or minimum of S. If S′ contains members, let y be the upper limit or maximum of S′. Then y is at least as great as any member of S′, and it follows, since we can easily show that S′ contains members which approach as closely as we choose to the members of S, that there are members of S smaller than $y+e$, however small a positive real number e may be. On the other hand, there can be no member of S smaller than y. For suppose that z is such a member of S. Then, in accordance with the way in which we have determined S′, since it is made up of all positive or zero real numbers less than every member of S, there can be no member of S′ greater than or equal to z: that is, there can be no member of S′ greater than or equal to $y—e$, where e is the positive number $y—z$. Consequently y fails to satisfy the definition of the upper limit or maximum of S′. But y is by definition the upper limit or maximum of S′, so that our supposition that there is a member of S smaller than y engenders a contradiction. We see as a result of this that y satisfies both of the conditions which go to make up the complete definition of the minimum or lower limit of S, so that there exists a minimum or lower limit of S in this case, as well as in that where S′ has no members. If we now consider the remaining alternative concerning the natures of S and S′—that is, if S contains no members whatever, as is the case when it is made up of all odd multiples of ten or of all integers that are commensurable with π—we shall, to simplify matters, make an *ad hoc* definition of the lower limit or minimum of S, and shall say that this lower limit or minimum is zero. This latter definition is, it is true, somewhat artificial. If we make this convention,

it follows from what we have said that any conceivable collection of numbers has at least one minimum or lower limit: the uniqueness of this limit, to which we have already referred, may be demonstrated by a very simple proof, quite analogous to that which we gave last time for the uniqueness of the *maximum or upper limit* of a class of positive numbers.

We have just seen how it is always possible to assign to a given set of positive or zero real numbers one and only one positive or zero real number which is smaller than or at least as small as any member of the set, but to which the members of the set approach as near as we please, and we have called this number the minimum or lower limit of the set. Let us see whether we can define the distance between two points as the minimum or lower limit of the set of values of I for different values of I for the various paths connecting the points in question. In the first place, we must show that, provided that there is a smallest value of I, this must be the minimum or lower limit of all the possible values of I for the two points, since this condition is necessary if we are to regard the distance between two points as the length of the shortest path between them, as we do in ordinary space. This is true because the smallest of a set of numbers is at least as small as any member of the set, while there is no degree of approximation with which you cannot make it represent some member of the set, since it is itself a member of the set; consequently the smallest of a set of numbers, provided there is such a number, is the minimum or lower limit of the set. It results from this that the length of the shortest path from A to B —i. e., the least value of I for the two points in question— which we should naturally call the true distance between A and B, if it exists, is precisely that lower limit or minimum of the possible values of I which we have just agreed to call the distance between A and B. However, if there

is no shortest distance between A and B along any path made up of the vectors that we have already measured—that is, if there is no least value of I—there must always, by what we have just seen, be a minimum or lower limit of the values of I, which will have many of the properties that are characteristic of a minimum value of I, and which will satisfy the definition which we have just formulated of the distance between A and B.

Given our sphere of comparison, then, we have thus been able to define in terms only of those fundamental notions which we have already explicitly formulated the distance between any two of those generalized points which correspond to ordinary geometrical points. Upon a slight investigation, we should find that we can prove, independently of any assumptions concerning the formal properties of the objects exemplifying our fundamental notions, a few simple geometrical theorems concerning distances. We can prove, for instance, that if A, B, and C are any three points, the sum of the distance from A to B and that from B to C is not less than that from A to C. We can also prove that the distance from A to B equals that from B to A. We cannot, however, prove from our definitions alone, that the distances that we have so far defined have *all* the formal properties of distances in ordinary geometry. In ordinary geometry, for example, if the mutual distances of four distinct points are given, and the distances of a fifth point from three of these are known, the distance of this fifth point from the remaining vertex of the tetrahedron formed by the four original points is determined to have one of only two possible values, while our definitions do not secure to any distance any precise quantitative relation to other distances, in general.

We have given the definition of distance just developed with the original intention of using it only for those distances that do not fit into our sphere of reference—the dia-

meter of which, by the way, is still the unit of our sphere of measurement. If, however, we examine into the definition, we see that it may also naturally be applied to distances which fit into our sphere of measurement—for any two generalized points which are separated by a vector lying in our sphere of measurement, and consequently by the extension of this vector, are thereby connected by a chain consisting of that single step, and consequently have a distance in the sense in which a distance is defined in terms of a chain of vectors. It is further obvious that in a system of generalized points which behave in a decent geometrical manner, a chain connecting A and B and consisting of a single vector is at least as short as any other chain connecting A and B, so that the magnitude of the vector from A to B is the same as the distance from A to B, in the sense of the minimum length of a chain from A to B. We can consequently throw away our first definition of the distance from A to B entirely, and be sure that if we define all distances in space by chains, our definition will be natural—at least as natural, that is, as our direct definition of distances by means of vectors. We have thus attained a definition of all the distances in space which involves besides the notions that we have taken as primitive only that of a certain standard sphere of measurement.

The question now arises, what sphere is the actual unit sphere of our measurements? If it turns out that we can select no set of generalized points as the unique unit sphere, how are we to determine the various spheres which shall serve as the bases of our measurements, and if these spheres give us different values as expressions of the magnitude of a given distance, how can we reduce the various results which we obtain to a single internally coherent system of measurement? It is easy to indicate two directions in which we may approach this problem, corresponding respectively to the two definitions which we gave of parallel

lines in a preceding lecture. It will be remembered that in the process of obtaining one of these definitions of parallel lines we defined a certain collection of sets of generalized points called a-spheres entirely in terms of our experience of the intersection of convex solids. We saw that if our experience of the intersection of convex solids records two convex solids as intersecting when and only when they both are situated in a certain region of space and approach one another closer than a certain distance which is the same everywhere in space, our a-spheres, *qua* sets of generalized points, will be spheres of a certain uniform size. This result will seem more familiar to you if I put it in another form. Many of you have seen the various processes of the differential and integral treated in the text-books of the more old-fashioned sort as if they dealt with operations of division or summation among certain entities called infinitesimals. These infinitesimals were regarded as magnitudes, all very small, but equal, in general. By the English mathematicians of the eighteenth century, who, like their philosophical colleagues, were of a more empiricist attitude to their subject than those of the Continent, these infinitesimals with which the calculus seems to deal were regarded as if they were the smallest objects accessible to our direct sensory experience. Since it was an essential property of the infinitesimals, in their mathematical use, that they should be equal, in general, those mathematicians who held this view were driven to the supposition that all just-noticeable sensible objects—all *minima sensibilia*—are equal. Now, it will be remembered that our a-spheres bear a very close analogy to the *minima sensibilia* of the philosophers, in that they mark the lower boundary of the sensibilia we are considering—namely, convex solids—with respect to their magnitude. Our assumption that all a-spheres are equal is consequently one of those things which was involved as an unquestioned presupposition in

the empiricist treatment of the calculus, although it involves the admission of far less than is necessary for the support of that view. The fact that it is such a natural hypothesis, and that it is an element in a view of the greatest historical importance, makes the consideration of a theory of measurement based on this hypothesis a thing of the utmost interest, whether it be strictly true or not that all *minima sensibilia* are what we should ordinarily regard as of equal magnitude. Let us first notice, however, that it is extremely improbable that the delicacy of our sensory discrimination, and as a corollary the size of our *minima sensibilia,* is in any ordinary sense the same throughout all those parts of space which are more directly accessible to our sense-experience.

On the assumption that we could naturally call all a-spheres equal, and that our space has all the normal geometrical properties, it is a matter of indifference which a-sphere we use as a unit of measurement, and all the diameters of all a-spheres are equal. It will not, however, be in general a matter of indifference which a-sphere we take as our standard sphere, if this assumption is not fulfilled. In such a case, it may be possible to get several numbers, all of which express with equal right the distance between two points, A and B, according to the a-sphere which we choose as our unit of measurement. If we are to be able to regard some single number as the only true distance AB, as measured in terms of the diameter common to all a-spheres as a unit, we must find some way of obtaining from all these different measures of this distance some single quantity, which is uniquely determined by all these various measures, and which coincides with their common value if they agree. There is, of course, a certain degree of arbitrariness in our selection of this distance, but one may easily show that if we regard the true distance AB as the minimum or lower limit of all the values which we can get for

the distance AB by using various a-spheres as standard spheres, we shall obtain an entirely unequivocal definition of the distance AB. In the first place, this distance will always exist, be finite, and uniquely determined, since, as we have seen, all these things are true of the minimum or lower limit of any set of positive or zero numbers. In the second place, if all the distances of A from B, measured by a-spheres, agree, they will all coincide with their minimum or lower limit, as one may see on inspection. We have thus obtained a definition of the distance between any two points in space which involves no notion other than that of our experience of the intersection of convex solids, which secures that any two points in space shall be at one and only one distance from one another, and which will completely agree with our usual notions of the distance between two points, provided that our experience of the intersection of convex solids has such properties as we should naturally expect it to have, and provided, moreover, that we can naturally call all *minima sensibilia* equal.

In the system of measurement thus obtained, some of the theorems of ordinary geometry will still hold good, irrespective of any assumptions about the nature of our fundamental experience; for example: the distance AB will equal the distance BA, the sum of the distances AB and BC will be at least as large as the distance AC, and so on indefinitely. For the most part, however, the geometrical properties of our distances will be dependent on the nature of our experience of the intersection of convex solids, and will not, in general, agree with those characteristics of ordinary Euclidean geometry, if our fundamental experience has not such properties as we should naturally expect it to have. For example, the theorem that, if any tetrahedron is given, the distance of a point from one of its corners must assume one of a certain pair of values, once the distance of the point from each of the other three

corners is known, which is true in ordinary geometry, cannot be deduced from our definition of distances alone.

However, the object of this course of lectures is to show how we can eliminate from geometry all presuppositions concerning the particular formal properties of our original notions, and yet define our points, lines, distances, etc., in terms of these in such a manner that all the ordinary formal geometrical properties of these latter entities should follow from their definitions alone. To do this, we must be able to obtain a definition of distance which will depend on our experience of the intersection of convex solids alone, and which will of itself be a sufficient guarantee that our distances satisfy the usual geometrical laws, and it may further be shown that, once we have a perfectly satisfactory definition of all the distances in space, we have it in our power to give perfectly satisfactory definitions of all geometrical entities. As we have just seen, we have not yet succeeded in completely performing this task; the problem, however, is an extremely interesting one, and one of the utmost philosophical importance. Let us turn our attention to it for a little while.

Our last definition of distance has practically indicated to us how we should make a survey of the universe, for it correlates with every pair of ordinary generalized points the distance between them. However, it will be a bad survey, in general. By this I mean distances which it causes to separate points will not check up, will not "gee" with one another. You all know that a surveyor always makes more observations than are merely sufficient to indicate the position of each point he is mapping a single time only: he observes the same point five or six times over from various observation-stations, and records all his observations. In an ideally perfect survey, all these observations would come out in complete accord with one another, but as a matter of fact, they seldom or never come out in com-

plete accord with one another in any actual survey. The several determinations of each triangulation-station indicate on the map, not one point, but a number of points, situated in more or less close proximity to one another. It is a part of the task of the surveyor to fix a single point, which may be said to be the best representative of these several points, in terms of the collection made up of all of them. This he does by means of a certain mathematical theory known as the theory of least squares. This theory does not concern us here, except in so far as it fulfills the function of reducing a set of unharmonious measurements to a harmonious system; we shall have nothing to say concerning its technical details. Its function is to make every single point which forms a station in a certain survey correspond to one point on the map representing the results of this survey, and to one only. Then the surveyor can go and take the distances between these uniquely determined points on his map, and say that they represent—when the appropriate alterations of scale are made—the distances between the points on the earth's surface corresponding to the points on the map, and we may be *a priori* sure that the proper geometrical relations will hold among the distances so determined.

In order to make the discussion of surveying which we have just given more simple to grasp, we have been guilty of a slight inaccuracy of statement. We said that the several determinations of each triangulation station indicate on the map, not one point, but a number of points. As a matter of fact, however, the several determinations of each triangulation-station indicate no specific point whatever on the map until we have already determined a certain correspondence between points on the map and points on the surface of the earth. This presupposes that we are able to locate certain points on the map before others and to discriminate between such as we shall locate first and

those which we locate later. We do not possess, however, any criterion for such a location. It is consequently necessary for us to possess some means of deriving at one blow from our confused measurements of the distances between our stations quantities which we can regard as the expression of the magnitudes of these distances, among which the proper geometrical relations will of necessity hold, if we are to be able to map our triangulation-stations in such a manner that to each triangulation station there will correspond one and only one point on the map. This task can also be accomplished with the aid of the method of least squares, in the case of any survey involving only *a finite number of points*. From any survey, then, however bad it may be, provided that it involves only a finite number of points, a set of finite quantities may be obtained, one of which is correlated with each pair of the points surveyed, and among these quantities the formulae which correlate distances in an ordinary Euclidean space will hold.

However, the bad survey of the universe which was made by our definition of distances in terms of a-spheres involves the measurement, not of a finite number of distances, but of an infinite number of distances, provided that the space that we have defined in terms of our experience of the intersection of convex solids is comparable in richness with the space of ordinary geometry. Now, the method of least squares is unable, as far as I know, to bring order into surveys that are bad at an infinite number of points. We do not at present possess a method of deriving a well-behaved Euclidean set of distances from the hodge-podge set of distances which we obtain, in general, from our definiton of distances in terms of a-spheres. I see no reason, however, for supposing that the problem of obtaining a method which should perform the same function for infinitely irregular systems that the theory of least squares fills for finitely irregular systems should be essen-

tially insoluble. The solution of this problem is of absolutely vital importance for the philosophy of space, for however we may define distance, nothing is more certain than that the distances of points in space, as we first learn and observe them, only approximately satisfy the laws of geometry, yet we are absolutely sure that there are actual distances which our observed distances represent and which satisfy these laws precisely. If we were enabled by a theory analogous to that of least squares, to derive from our chaotic observed distances, distances whose geometrical properties and relations should be secured *a priori,* we could immediately explain this phenomenon, on which so much importance has been laid by philosophers.

In this lecture, we have extended the system of measurement determined by a sphere from its inferior to the whole of space. On the basis of that definition of parallelism which starts with a-spheres, we saw that a system of measurement could be developed, but that this would have certain defects. We saw that this system of measurement does not secure *a priori* the geometrical properties of space, but that it would only require the formation of a branch of mathematics which should deal with problems essentially similar in character to those dealt with in an already existing branch to give us a space whose geometrical properties would be *a priori.* In our next lecture we shall take up the problem of defining distances on the basis of our alternative definition of parallel lines, and we shall finally discuss the philosophic problems to which both these systems of measurement give rise.

VIII. Geometry an a Priori Science

IN OUR last lecture, we developed a theory of measurement upon the basis of that definition of parallel lines which involved the notion of a-spheres. In this lecture, we shall first develop an alternative theory of measurement, in which we follow out that definiton of parallelism which involved, not only the relation of apparent intersection among convex solids, but also four selected convex solids, which we agreed to call "unit spheres." If the distances of any point in space from the centers of these four spheres are known, and our four spheres, to put it in ordinary geometrical terms, are four spheres whose centers are not all coplanar, the position in space of the point in question is completely determined, and if the positions of two points are determined in this manner, their distance from one another is completely determined. As we intend in what follows to secure the proper geometrical interrelations of our distances, and to cause the theorems of geometry to be satisfied, by means of defining the position of any point in space in terms of its distances from the four centers of our standard spheres, the first notion which we must define is that of the distance of a point from the center of a sphere. Now, it is obvious that, whether a point x is inside a sphere S or not, the furthest point in S from x lies on the line through x passing through the center of S, and is that one of the points on the surface of S and also on the line in question which is further from x. If y be the point in S furthest from x, it therefore follows that the distance from x to y is the distance from x to the center of S, plus the radius of S. One might consequently think that we

might define the distance of x from the center of S as the distance of x from the most remote point of S, minus the radius of the sphere. We wish, however, to have our definition applicable even where S is not precisely a sphere, and we consequently do not wish to make the assumption that there is any single point of S most remote from x an essential condition of the usefulness of our definition. Now, if there is any single point in S remote from x, the distance of this point from x will be the maximum or upper limit of the distances of points in S from x. However, it will be much more usual, in general, for there to exist a maximum or upper limit of the distances of points in S from x than for there to exist some single point at precisely that distance from x and belonging to S. We shall consequently regard the distance of a point from the center of S as the maximum or upper limit of the distances of points in S from the point in question, minus the radius of S. Now, if we make S itself our standard sphere of measurement, in the sense explained in our last two lectures, since the diameter of S will be our unit, the radius of S will be one-half. Consequently, it will be natural to regard the distance of a point from the center of S as the maximum or upper limit of the distances of points in S from the point in question, minus one-half. This definition, as may be readily seen, involves no other notions than such as we have already taken explicitly as primitive, together with such notions as "one-half," or "limit," which, as the modern logicians have shown, are purely logical notions.

Given the four equal, non-coplanar spheres that served as the foundation of our second theory of parallelism, we are now able to define the distances of any point we please from the center of each of them, in terms only of such notions as we have already taken explicitly as primitive. It is interesting to note that our definition involves absolutely no reference to such entities as the centers of these

spheres. Be that as it may, we can treat our definitions as if they yielded us the distances of a point in space from four given points in space—the centers of our four spheres. We intend to use these four distances as the coördinates of our points, and to determine the position of any point in space by means of these four distances. We finally desire to determine the distances of any two points in space from one another in terms of the distances of each of the points from each of the four centers of our equal spheres. Our actual coördinates will not, however, be the four distances of a point from our four centers, since we wish our coördinates to be absolutely independent of one another, as this will facilitate such a definition of the distances in space as will secure automatically their satisfaction of the laws that bind distances to one another in ordinary geometry. Now, in ordinary Euclidean space, if we know the distance of a point from each of the vertices of a triangle, we know that its distance from any fourth point in space can assume one of only two possible values. We shall consequently determine a point, not by its distance from each of our four centers, but by its distance from three of our centers, and by whether its distance from the fourth center has its greatest possible value, the other three distances being given, or not. We shall, for example, write the coördinates of a point whose distance from the center of the standard sphere A is *a*, whose distance from the center of the standard sphere B is *b*, whose distance from the center of the standard sphere C is *c*, and whose distance from the center of the standard sphere D has its maximum value, *a*, *b*, and *c* being fixed, in the form $(a, b, c, +)$, while if the distance of our point from the center of D has not its maximum value, and everything else remained unchanged, we should represent our point by $(a, b, c, -)$. In these definitions, we have introduced no notion that we have not already taken explicitly as primitive.

The three coördinates and a sign by which we determine a point are completely independent of one another if our space already obeys the laws of ordinary geometry. If, however, we know the coördinates of two points, we do not yet know, by these data alone, the distance by which the two points are separated. This knowledge depends further upon a knowledge of the shape and size of the tetrahedron formed by the four centers of our spheres of reference—that is, on a knowledge of the remoteness of the four centers from one another. To discover this, we must already possess a definition of the distance between the centers of two unit spheres. A definition which is in every way analogous to the definition which we have already given of the distance of a point from the center of a sphere reads as follows: the distance between the centers of two spheres, S and T, is the maximum or upper limit of the distances between points in S and points in T, whether measured with reference to S or to T as the standard sphere, minus the radius of S plus the radius of T, which is one, since the diameters of S and of T are regarded as possessing a common value, and this common value is taken as our unit of distance.

We now possess sufficient data to transform the system of coördinates which we have already obtained into an ordinary Cartesian system of coördinates. If we form certain functions of the coördinates already defined for a point *p* in a perfectly determinate manner—which, it is true, are a little too intricate for us to exhibit here—and call these X, Y, and Z, we may be sure that, if our points, lines, distances, etc., have such properties as we should naturally associate with the names we give them; and if our four "spheres" are actually four equal spheres whose centers do not all lie in the same plane, our coördinates X, Y, and Z, will be an ordinary set of rectangular Cartesian coördinates whose axes are determined in a certain definite man-

ner by our four spheres of reference, and whose unit of distance is the common diameter of all four standard spheres. We may also be sure that even if our space is not so completely subject to the ordinary laws of geometry, many of our points will, in general, still determine coördinates, and any point that determines the coördinates X, Y, and Z will determine them uniquely. We shall call the values of X, Y, and Z that are determined by a certain point p the *fundamental coördinates* of p.

It is by no means a necessary consequence of the definition of the fundamental coördinates of a generalized point that to a given point there must always correspond some set of coördinates. Let us call those generalized points that have fundamental coördinates *proper* generalized points. By the definition of a proper generalized point, it must have one set of fundamental coördinates and one only. It does not follow from this, however, or from anything else we have yet said, that no two distinct proper generalized points have the same set of fundamental coördinates, whatever the formal properties of our fundamental notions may be. We desire, however, to obtain ultimately a definition of a spatial point which will secure by itself that no two points hold the same position in space—that is, that no two points have the same fundamental coördinates. How shall we do this? We shall do it by introducing a new definition of a point in place of our definition of a generalized point, just as we formerly introduced the definition of a generalized point in place of our very first definition of a point. Our final definition reads as follows: a *revised point*, we shall say, is a class other than the null class, or class without any members—of all the proper generalized points that have a given set of fundamental coördinates. Obviously, we assign this set of coördinates to the revised point in question. It is evident, then, that no two distinct revised points can have

the same set of coördinates, for otherwise they would coincide. A revised point, then, must determine a set of coördinates different from that of any other revised point; it further follows from the mode of formation of a revised point that every revised point has a set of coördinates, and that no revised point can have more than one set of coördinates. The appropriateness of calling revised points points will become obvious if you reflect that in an ordinary, well-behaved space a revised point will contain only one member —a certain generalized point—and that the revised point will consequently represent the same point, in our everyday sense of the term (whatever that sense may be) as the single generalized point which is its member. We have thus obtained a set of entities which can naturally be called points, between which and certain sets of coördinates or triads of numbers there subsists a certain correlation which is one-one, and extends over all of our revised points. In ordinary geometry, the correlation between points and their Cartesian coördinates is not only one-one, but also connects all the points in space with all possible sets of coördinates. Consequently, if our revised points are to have all the formal properties of the points in ordinary geometry, we must show, over and above what has been already shown, that every set of Cartesian coördinates—every triad of real numbers—determines some point. Now, it does not follow from the definition of a revised point that we have just given that every set of Cartesian coördinates determines a point. The manifest and obvious way to remove this imperfection in our system of definition is to find some entities that will fill the gaps in our system of points which are left by the absence of a revised point corresponding to a given set of coördinates. We can do this in the following manner: if to a given set of coördinates there corresponds a revised point, we shall say that the revised point is the point, in our final sense of the word,

that corresponds to it, but if there is no revised point corresponding to a given set of coördinates, we shall call the set of coördinates itself the point that fills the position indicated by our set of coördinates. If we thus interpolate sets of coördinates as points into our system, we shall find that our complete set of points will be in one-one correspondence with our complete set of coördinate triads. We shall thus have obtained a space which agrees perfectly in this respect with the space of ordinary geometry.

Now, it is familiar to all those of you that have had an elementary mathematical training that all the theorems of geometry my be reduced to purely algebraic theorems, entirely independent of space and dependent only on the properties of number and quantity, when once a system of Cartesian coördinates has been set up. It has been shown by the modern mathematical logicians that all theorems that deal with number and quantity alone are theorems of pure logic, and are independent of any concrete experience whatever. Since these things are the case, then once a system of Cartesian coördinates has been defined in terms of our fundamental notions, we have done all that is necessary to prove the *a priori* character of geometry, for we can now so define lines, planes, circles, angles, etc., in terms of our set of coördinates—and ultimately in terms of those notions that we have explicitly taken as primitive —that all the theorems of geometry shall result from these definitions and the laws of algebra alone. Consequently, if the space of our every-day life is constituted of the entities that I have just called revised points and of the number-triads of the space that I have just defined, the theorems of geometry are *a priori* true, even though space is actually a function of experience.

This brings me to the end of the technically logical and mathematical portion of the present course. Let us consider what we have accomplished, and let us see in how

far we have fulfilled the purpose which this course was designed to fulfill, and have proved or rendered more probable the thesis which we set out to prove. The thesis of this course of lectures is that, whereas space is a function of experience, the geometrical properties of space are *a priori* certain, and may be proved without involving any reference to the concrete nature of the experience of which space is a function. We claimed that the geometrical properties of space were due to the method of schematization by which space is obtained as a function of experience. To show that such a situation is possible, one thing that must be done is to exhibit a system in which geometrical properties are the results of a method of schematization applied to an arbitrary subject-matter, and of this method of schematization alone. We can fairly claim to have accomplished this task by exhibiting the spatial system resulting from our second definition of parallelism, for in this we have introduced no notion pertaining to a certain spatial system—no notion, indeed, other than that of a certain relation, which we *call* the relation of experienced intersection among convex solids, and four arbitrary terms or sets of terms that enter into this relation, which we call four equal spheres whose centers do not all lie in a single plane—and notwithstanding the arbitrariness of the formal properties of our fundamental notions, we have so framed our definitions that certain of our entities must of necessity possess the formal properties characteristic of geometrical objects. We can further claim that we have pointed out that it would not require the solution of any problems generally different in character from those solved in already existing branches of mathematics (such as the theory of least squares), to derive a system of entities obeying all the formal properties of geometrical entities from certain relations or facts essentially similar to those more or less directly accessible to our experience, in such a manner

that no presuppositions concerning the formal properties of these relations and facts are involved. That is, we have made it seem very probable that, even if we did already possess a geometry, and had no knowledge whatever of points, lines, planes, etc., we could build up from those facts ascertainable by means of a more or less direct experience, by a method involving only such notions as belong to logic and are not dependent on any concrete experience, a system of entities which one could naturally call lines, points, planes, etc., in such a manner that one could be *a priori* sure that the formal properties of these lines, points, planes, etc., would be those laid down by the laws of ordinary geometry. All this, though suggestive of the actual relations that subsist between geometry and experience, is not in any way conclusive evidence as to the nature of these relations. We have given no reason to justify us in supposing that the entities that we have called points, lines, planes, etc., are the same entities that we call by those names in our every-day life. Indeed, the feeling of artificiality that haunts us at every step of the ground we have covered seems to forbid this view, and I think that one will be perfectly justified if he categorically denies that the particular entities that we have called by various geometrical names are those that are called by those names in our every-day life. But the fact that there is nothing inherently impossible in the formatioin of a space whose geometrical properties shall be *a priori* certain, yet which will be a function of experience alone, and the absence of any other existing view of the relation between space and experience that will explain the association of certain geometrical entities with certain empirically known physical entities, and in addition thereto the non-experimental nature of geometry, entitles us to say that we have rendered the view that geometry deals with some schematization, some systematization, some arrangement of experi-

ence highly probable. We have given good reasons for supposing that the apriority of geometry is genuine, but that it is an apriority of method, not of subject-matter, and we have illustrated how an apriority of method which is not apriority of subject-matter is possible in the field of geometry.

We have the question still before us: if space, as we have said, is a schematization of experience, what sort of a schematization is it actually, and how is this schematization related to that which we have exhibited in this course of lectures? The first thing to notice is that it is very highly probable that the schematization by which the space of our every-day life is formed is probably not a fixed, immutable schematism, for which there is one analysis that is always right, while all the other accounts of its structure are always wrong. For instance, the schematization by which the space of a carpenter is reached is necessarily different from that by which a physicist attains his space: one might almost say that with the physicist, his straight lines are his light-rays, for a light-ray in a vacuum is the criterion by which he tests the straightness of anything else, and consequently it is at least reasonable to suppose that the complicated synthesis and organization of experience through which he must go in order to obtain a light-ray—since light-rays, as such, are not given in experience—must play a part in the construction of the lines of his geometry; while this obviously cannot be the case with the carpenter, whose criteria of straightness are simply chalk-lines and T-squares. It seems likely, moreover, that even with one and the same person, the schematism he uses may vary with the problem which he is attacking: on one occasion, the physicist may find it more convenient to regard a point as if it were built up by some process of organization similar to that by which he organizes his experiences under the form of a light-ray; on another

occasion, his method of schematization may be more analogous to that by which he obtains a gravitational line of force. It may be incorrect to speak of "space" as something unique: all that we ought to mention may be "spaces."

On the other hand, I feel fairly sure that certain tasks will have to be performed by all the divers methods of schematization that may lead us from experience to space. For example, the problem to which we devoted our third and fourth lectures—the problem, namely, of proceeding beyond that portion of space which is more or less directly accessible to our experience, and of obtaining definitions of points and of lines which will yield us all the points and lines in space—is one that is inseparable from any method of schematization by which we obtain space from experience, and I believe that the method of making this extension of space that we there developed is essentially similar to that which we use in our actual processes of obtaining space. The problem of an infinite theory of least squares, of turning a bad survey of the universe into a good one, is one that we must meet in almost any process of deriving space from experience. The problem of distinguishing parallel lines from intersecting lines is another problem that is not confined to the system developed in this course. It will thus be true that though there are many different systems of schematization which, on certain occasions, yield us space as a function of experience, and while the method which we have described in the preceding chapters may be different from all of these, there will be certain problems that run through all these methods and the one which we have given, so that an exposition of the method which we have given cannot but throw light on the methods of schematization by which space is actually obtained.

It may seem to you that these tasks which I have just mentioned as essential steps in the attainment of space as a function of experience are, as our introspection shows

us, not performed consciously, while they are too intricate to be performed unconsciously. I think, however, that both of these statements are open to question. In the first place, whenever a man thinks clearly enough to give a definite criterion for the straightness of a line, or the intersection of two lines, or whatever other geometrical property you please of physical, sensible objects, he is really simply rendering explicit some stage in the synthesis of experience which constitutes his space, and in so far as he has succeeded in forming his criteria of straightness, intersection, etc., into a system which must, by an internal necessity, be coherent, he has succeeded in making the complete method of schematization of his space determinate. We have already referred in this course of lectures to the fact that the surveyor, when he obtains a coherent map by a definite method from a mass of disharmonious data, is doing something of essentially the same nature as what we have been trying to do in this course of lectures, for he possesses a method by which he can deduce a map in which the laws of geometry hold from the most hopelessly incoherent and lawless set of observations or experiences. In the second place, it is a well-known fact that the complexity of a mental process is in itself no absolute bar to its unconsciousness. A man may have the most complicated set of criteria by which he determines the straightness of a given line or the flatness of a surface, but he may never have introspectively considered the nature of this process. It may turn out that many or all of the geometrical properties of his space result from these criteria alone, and that these criteria or definitions of straightness, flatness, etc., are so formulated that they constitute a perfectly coherent system, without his ever explicitly knowing that he uses these criteria or definitions at all. It does not seem to me at all unlikely that the mathematician or physicist, who unconsciously performs such intricate processes

of reasoning as differentiation or integration, should perform unconsciously some comparatively simple synthesis whereby space, with all its geometrical properties, is obtained as a function of experience. As to the non-mathematician, who is unable to follow a complicated train of reasoning even consciously, and who can formulate only vague and unclear definitions, there is no reason to suppose that in the space which he obtains as a result of his own process of synthesis from experience the laws of geometry hold in any but a rough and vague and rough way. In short, it is perfectly possible for the methods of schematization by which space may be obtained from experience to be unconscious, at least in the sense that we never see it as a systematic whole.

In closing this course, I wish to make a few remarks about the manner in which the theory of space we have developed has answered that problem which we found the Kantian view of space unable to meet. I refer to the problem of the correspondence of certain physical objects with certain spatial entities. How can we use one of the geometrical lines that we might define in terms of our revised points, for instance, as a criterion of the straightness of such a convex solid as a mark on the blackboard with a piece of chalk may be? The answer to this question is very simple. We have already seen how we can regard a convex solid as a set of points, in our first sense, and we have also seen how we may make a set of points in our first sense determine uniquely a set of generalized points. We are thus able to regard a convex solid as if it were a certain set of generalized points. It is easy to carry this process a little further, and to regard a convex solid as a certain set of revised points determined uniquely by it, which may be regarded as the set of all the revised points inside the convex solid in question. We can then give a simple mathematical definition for the accur-

acy with which the convex solid in question represents a straight line: we can say, for example, that the linearity of a set of points is the ratio of the longest linear segment connecting two points of the region or set to the longest perpendicular segment that contains two points of the set. On Kant's view, it is essentially impossible that anything of the sort should be done, since his lines, forming portions of space, can be given to us only *a priori*, so that we cannot recognize them in an empirical situation, and associate them in a definite manner with empirical objects. We have thus completely established our case against Kant; our case against Mach and the whole Empiricist school of philosophers of mathematics has already been made out.

NORBERT WIENER.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

THE FAILURE OF CRITICAL REALISM

EX PEDE HERCULEM. A realism in which perception, as such, is incapable of ever apprehending material existents *qua* existents is—obviously—completely debarred from direct awareness of the physical Universe. It may, by means either of explicit arguments or of instinctive and irresistible belief, posit the reality of such a Universe; but only as a World with which the knower can never come into absolutely primal contact and direct relation. Thus the crucial test of realist epistemology is its treatment of the process and content of perception. All the later stages in the development of knowledge—thought, universals, mind—though of more vital importance to philosophy, are nevertheless very largely determined by the conclusions of the prior inquiry into perception. If then the content apprehended in and through perception is never under any circumstances ontologically identical with the material world, realism degenerates into Noümenalism.

This appears to me to be the logical outcome, on its own confession, of the noetic system developed in *Essays in Critical Realism.*[1] It concludes with a "lack of absolute certainty"—a lack not merely occasional but eternal; "what we contemplate is, in the case of perception, *appar-*

[1] Macmillan, 1920. For the sake of brevity I have treated the volume as one whole, without explicit reference to the various authors, except in the few cases in which purely individual opinions are advanced. It may be noted here that perception occupies a "fundamental position in any theory of knowledge," (p. 97).

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ently the very physical object itself."[2] But—*semper, ubique et ab omnibus—only* apparently; for "this outer existent is not literally grasped; only its what, essence, character is grasped; our knowledge is obviously fallible";[3] and so far as perception, the root of knowledge, carries us, not only sometimes, but always, fallible. Perception is *la connaissance—la verité—manquée.*

This result springs from a distinction which, in spite of some ambiguity, is presented by Professor Drake as fundamental—the distinction, *i. e.*, between "characteristics of objects" and "objects themselves." "Characteristics appear to us; objects themselves," on the other hand, "do not get within our consciousness; the physical existent itself does not get within experience; knowledge is a beholding of (its) what, its nature."[4] But since "objects themselves" are indubitably real—for otherwise ontological realism vanishes—and since further our instinctive belief in their reality is well founded—for if not, then epistemological realism vanishes also—"objects" and their "characteristics" must constitute, in virtue of their dual relation to consciousness, experience or knowledge, two separate categories. The existence of "objects themselves is private, incommunicable";[5] what is known is their characteristics, essence, nature.

This is the case if we take the passage as it stands; but if we compare it with page four, there is some self-contradiction. For (p. 24), "we directly perceive . . . the character of objects"; on the other hand (p. 4), "what we perceive . . . is the outer object itself." Taking these two statements together, then, "objects" and "character of

[2] The word "object" bears a special meaning; *cf.* p. 213. I have therefore used "thing" or "existent" in referring to physical or material objects in the ordinary sense.

[3] Pp. 32, 20.

[4] Pp. 24, 29.

[5] P. 24. *Cf.* p. 240: "The physical thing and the psychic state . . . are unquestionably two and mutually independent." I may refer to a previous discussion of Dr. Strong's theory in *The Journal of Philosophy*, Vol. XVI, p. 428.

objects" are, for perception, identical, and the distinction I have just referred to disappears completely. If, indeed, we confine ourselves, as the volume before us explicitly does, to the epistemological standpoint, it is difficult to conceive how any such distinction between objects and characteristics can ever arise. Unless the content of our knowledge of the world, however extensive and abstract it may in the end become, is based ultimately on the content perceived, it can arise in some way or other only from the activity of the mind; and when this possibility is excluded, noümenalism can be avoided only by identifying the perceived content ontologically, to some extent at least, with physical things themselves, which then become (again to some extent) one with their "characteristics."

"To some extent," that is; for this principle precludes neither the logical grading of the essentiality of the thing's characteristics, nor the ascription to it, for certain limited purposes, of attributes which are scarcely essential at all; as, *e. g.*, in poetry or metaphor or scientific hypothesis. But just as the issue of paper currency must repose on a gold reserve, so all such mental or knowledge processes must be based ultimately on an ontological core of reality directly perceived—a core or nucleus which is surprisingly small, but which is none the less indispensable; as, once more, a very restricted gold reserve would serve to maintain, under an ideal economic system, a disproportionately large credit system. If, on the other hand, perceived content is confined to characteristics which are always distinct from real things themselves, then the "gold reserve" of knowledge becomes wholly mythical, or it is at best locked up in an invulnerable safe the combination which is not only unknown but unknowable.

It is to this result that the epistemology of Critical Realism leads; for if we consider the main trend of the volume, the ambiguity referred to may be disregarded. There is

a universal, fundamental, distinction between "actual characteristics of objects" and "objects themselves"; it is no mere question of degree, of more or less, of extent and inclusiveness. The crucial passage in this respect occurs on page twenty-four: "So far as perception gives us accurate knowledge, it does so by causing the actual characteristics of objects to appear to us. The objects themselves . . . do not get within our consciousness." It becomes necessary therefore to examine the ground for asserting the real existence of objects, as distinct from their apparent, or perceived, or known, characteristics.

The assertion, we find, rests on an instinctive feeling or belief. "We instinctively feel these appearances to be the characters of real objects. . . . We may consider our instinctive and actually unescapable belief justified."[7] This standpoint undoubtedly expresses a most important principle; but its true significance demands careful analysis. For there is a sense in which all our beliefs and philosophical conclusions are unescapable, and (from that point of view) instinctive. Whatever we really believe, we believe because, in the end, we can not help so doing; there is always some ground, logical or otherwise, which compels us to modify or abandon hitherto accepted conclusions; no one seriously adopts a new position except on grounds which—whatever their precise nature may be—he feels to be irresistible. But it is the peculiar task of philosophy to criticize such bases for belief; and having once undertaken this examination philosophy can not, *as philosophy*, rest finally upon any basis which is merely instinctive and nothing more. This does not mean that an instinctive belief is never satisfactory and must never be entertained; if that were the case life would at once lose many of its

[6] I do not see how perception can be said, in any literal sense, to cause anything essential; the phraseology appears rather unfortunate.

[7] P. 6. *Cf.* p. 195, "our instinctive assertion of" the physical world, which we "affirm through the pressure and suggestion of experience." See also note 11 below.

most precious elements; it simply implies that it is never satisfactory *philosophically,* and can never be accepted as a final *philosophic* deliverance. There is a whole universe of difference between an instinctive belief accepted purely as instinctive, and an instinctive belief for which some non-instinctive basis can be exhibited; the second may be philosophical, but the first can never be so.

Of this principle the varieties of belief in the real existence of physical things is an outstanding example. Everyone begins with a purely instinctive belief in their reality;[8] but in the case of the "average man" or "naïve realist" this has become more or less rationalized, in so far as he bases his belief upon his "senses," which he means perception. And every form of Realism merely carries farther —it does not reject or subvert, as does, *e. g.,* subjectivism or solipsism—the critical analysis thus initiated in ordinary experience;[9] no other course is indeed open to it, unless it rejects perception and its content *in toto.* For (to revert to our illustration) while the naïve realist far overestimates the amount of his gold reserve, so that much of what he accepts must be rejected as spurious or as impure, none the less must some minimum of precious metal be preserved intact as the ultimate basis of the whole system of knowledge. All Realisms, in other words, must rest finally—exactly as does naïve Realism—upon perception and its content[10]—upon a content more deeply criticized and more rigidly tested; and this analysis must be carried through to a final verdict. Either this content is—as naïve Realism regards it—itself ultimate physical reality, although the conditions governing both its reality and our knowledge of it are more truly defined; or it is not ultimate physical reality. If then this negative standpoint is adopted, two further alternatives arise. Either ultimate physical reality is

[8] *Cf.* James, *Principles of Psychology,* Vol. II, p. 287.

[9] *Cf.* "Critical realism . . . is a criticism of naïve realism." *Op. cit.,* p. 189; and p. 196.

never ontologically identical with the content of perception, and then—since there is no other mode of knowing it directly—this reality is noümenal; or we fall back once more—at the end as at the beginning—on an instinctive, but non-philosophical, belief in the known existence of physical reality. This in itself is never presented as the content of perception; in no other way is it possible for it to appear to us; and still we believe in its known existence instinctively—simply and solely, *i. e.*, because we can not help doing so.

This is the dilemma which faces, it seems to me, Critical Realism. If it maintains its universal distinction between physical things themselves beyond our consciousness, and their perceived or apparent characteristics, then (as I have argued throughout) it becomes a noümenalism. But if on the other hand it founds its affirmation of the known existence of physical reality on instinctive belief, then it forfeits all title to be regarded as a philosophic system, despite whatever other merits it may possess.[11] Or, if it still claims to be such, it can be at its best only a philosophy of the content of perception—this content being, confessedly, always distinct from ultimate physical reality itself.

It is true that the critical realist, despite this distinction, claims that perception is directly of things (or objects) and not merely of content. "He sees no reason why (the object of perception) should not be called the direct object" (p. 103). But in thus insisting on the directness of perception, it is only in precisely the same sense that memory, thought and conception are direct; in this respect these diverse processes or activities are all alike; "the principle is not different in perception." Such a standpoint, however, seems to rest upon a fundamental misinterpreta-

[10] Adopting the usual distinction between "perception" as a process and "percept" as content.

tion of the relation between these purely ideational processes and perception itself. For while these non-perceptual functions are undeniably direct, still their directness is in its nature far removed from the immediacy of perception, inasmuch as the former necessarily succeed and are based on the latter, and always, implicitly or explicitly, refer back to it for their own substantiation. They can not sustain a conflict with veridical perception, however remote and abstract may be the ideal content wherewith they supplement it. *If* perception is direct, then memory and thought—so far as they remain concerned with the same objects and are free from error—will be direct also. But their immediacy is the result or consequent of the prior immediacy of perception; it is wholly illogical therefore to adduce it as evidence for its own basis; the argument is obviously circular and assumes the very point at issue.

Ideation then always refers back to perception.[12] Its directness is really of a secondary order as compared with the primary immediacy of perception. What then of perception itself? Does it repose upon any cognitive function or activity more fundamental than itself? Naïve realism replies in the negative; for it perception, even when corrected and supplemented by memory, conception, and thoughts, find its ultimate guarantee in further perception and in that alone. The standpoint of the natural sciences, in their relation to physical reality, is the same. In the end, and so far as it is at all possible, science also appeals to observation and experiment—that is to perception characterized by the utmost delicacy and exactness. This principle is in no way contradicted by the use of scientific concepts. For these, when they transcend the abstractness of

[11] Historically, this standpoint closely resembles that of Stoic philosophy, which regarded some "phantasies"—the Greek equivalent of our sense-data—as creating an irresistible belief, not only in their own existence, but in that of their external objective causes..

[12] *Cf.* "Realism takes its start from perception. . . . I have dwelt upon perception because of its fundamental position in any theory of knowledge." *Op. cit.*, pp. 89, 97.

pure mathematics and logic, are derived, and indeed can only be derived, from the prior content of sense-perception. In this respect there has recently occurred a marked change in the attitude of scientific investigators. Molecules, atoms and electrons—perhaps even the aether—are no longer universally regarded as unreal abstractions or as mere "conceptual formulae of calculation."[13] Both in themselves and in their spatial arrangements they are now placed, by the physicists most closely concerned with the investigation of their properties, on the same footing and in the same existential category as the common objects of everyday experience. This holds true even of the recent developments in the physical theory of relativity, despite its extreme abstraction and abstruseness. "The conception simultaneous . . . does not exist for the physicist until he has the possibility of discovering whether or not it is fulfilled as an actual case . . . (until) he can decide by experiment";[14] that is again, from the standpoint of epistemology, by perception. This is the case then so far as naïve realism and physical science are concerned; while if we turn in the opposite direction and trace perception, as so many psychologists do, to some origin in mere sensational content the result is pure subjectivism—the very antithesis of all realism.

For naïve realism, further, this finality of perception is absolute—or very nearly so; and in this, of course, lies its patent defectiveness. *All* that it contains or reveals or yields to knowledge is, with but slight exception and qualification, ontologically and existentially identical with physical reality; even the modern physicist does not hesitate to assume that with sufficiently heightened powers of vision, either natural or artificial, what we should actually see would be the real molecule or atom itself. *Everything* cir-

[13] *Cf.* Dr. A. N. Whitehead's recent volume, *The Concept of Nature*, Chaps. 1, 2; also *Nature*, Nov. 6, 1919, p. 230.
[14] Einstein, *Theory of Relativity*, p. 22.

culates, we may say, in gold currency, except for a few spurious coins and the minimum of paper. But in this respect naïve realism is obviously far too sanguine; investigation quickly detects a large admixture of mere tokens and bank notes as it were; part of least of what perception yields is far from being the identical physical thing itself; naïve realism therefore must be supplanted by some more refined type of epistemological theory.

This leads us to what is, in my opinion, the radical defect of Critical Realism. Just as naïve realism is too absolute in one direction, so Critical Realism goes to too great an extreme in the other. It carries its critical process—in principle perfectly legitimate—altogether too far; so far indeed that it defeats its own aim and undermines its own basis. It saws away the branch which supports it and so falls headlong into noümenalism.

For *every* element which was identified by the naïve realist with physical reality now becomes sharply distinguished from that reality and is regarded as "content"—content which means or indicates or even reproduces physical reality,[15] but which can never be, under any circumstances whatever, existentially identical with it. Of the perceived characteristics regarded originally as existentially inherent in the physical thing not a single one remains—they are all transferred to the category of datum or content, and are therefore wholly distinct from the thing itself. Every element, regarded as a physical quality of the material thing, is thus deprived of its existential physical status and becomes a datum. Every coin, in other words, tendered for examination by the naïve realist is degraded into a mere token. Certainly by its means his activities can go on quite as well as before; but neither singly nor in their totality are these tokens ever identical

[15] "The content which we apprehend must have the property of reproducing something about the object, of conveying in its own medium the form of the object." P. 218.

with the gold currency which he believed himself to be handling. He is, however, not a cent the poorer; his wealth remains there intact, to be just as readily used and computed, if he will but recognize that this can be done only by means of scrip instead of gold. He is given, in short, securities which are absolutely sound, but which are also absolutely irredeemable.

The insistence on the non-physical character of all perceptual content is indeed so emphatic as almost to amount to sheer subjectivism. It is, in the first place, "impossible to identify either the datum or the images . . . with the object. . . . The quality-group found in perception is not physical."[16] And it is extremely interesting to trace the manner in which, as the development of Critical Realism proceeds, this merely negative description of the quality-group as non-physical imperceptibly changes into its definite and positive designation as subjective; so that the philosopher, in his dread of the Scylla of naïve realism, is inevitably overwhelmed in the Charybdis of subjectivism, from which he attempts to escape on a crazy raft of representationism.

The prominence thus attained by these subjective and representative elements in the entire system is sufficiently evident in the following passages; and if it be objected that these are isolated and severed from their context, the reply is that they are cumulative, and that nothing in the context qualifies their definiteness. We find then that "the content with which we automatically clothe these (physical) realities is subjective . . . Knowledge is the insight into the nature of the object that is made possible by the contents which reflect it in consciousness."[17] Here is the dawn of representativeness—the object is reflected in consciousness; after which, *facilis descensus Averno*. For "what

[16] P. 96.

[17] Pp. 197, 200. *Cf.* p. 212, "the content is mental," and p. 240, "psychic state or sensation."

appear to us as physical things are in themselves of psychic nature," and "experience indicates an actual, causally-based agreement between the physical existent perceived and the content of perception. . . . The organism has perfected the agreement between the subjective datum and the object of perception";[18] and thus the earlier metaphor of "reflection" has given place to a more definite and intelligible "agreement." Finally, and almost literally,

> Last scene of all,
> That ends this strange eventful history,
> Is second childishness and mere oblivion,
> Sans teeth, sans eyes, sans taste, sans everything.

For "the knower is confined to the (subjective) datum, and can never literally inspect the existent which he affirms and claims to know."[19] How, in that case, he can ever cognize the "agreement" asserted must remain a mystery; "the situation is unique," continued Professor Sellars, whose position here is stated most explicitly: *"Internally,* or *in the percipient* himself, we have the content of perception."[20]

Thus it becomes perfectly clear that all the elements within his perceptual field which are regarded by the naïve realist as existential constituents of the physical thing, or evens as in their totality constituting that thing, become transferred, without any exception whatever, to the category of non-physical content—content which is, further, subjective, internal, and in one way or another representative or reproductive of the thing itself. This is, I think, an accurate summary of the argument thus far; and taking it as a whole, we are irresistibly reminded of the conclusion of the classic encounter between Alice and the Cheshire Cat. It will be recalled that the Cheshire Cat slowly faded away, till finally nothing remained within Alice's perceptual field

[18] Pp. 202, 203, 240. *Cf.* p. 218, "the content must have the property of reproducing something about the object."

[19] P. 203.

[20] P. 196: italics mine.

except its grin; and in precisely the same way does the physical thing lose all its perceived characters (as physical) so that they are all converted into subjective, internal, content; there is not even a grin left. "I have seen" said Alice, who was of course a naïve realist, "a cat without a grin, but never a grin without a cat." We are not told what become of the vanished animal; but if a critical realist had been present, he would have assured Alice not only that the cat still existed, but also—what is here the fundamental point—that she still continued to perceived it exacly as before! For the conversion into content in no degree interferes with perception; it is rather the means by which it is effected. "I perceive" maintains Professor Sellars, "concrete *things* . . . co-real with the percipient, and independent of him in exactly the same way and to the same degree that they are independent of one another. . . . Thinghood and perception go together";[21] and as we have seen already, Professor Pratt insists that perception remains direct.

The terminology here is of fundamental importance. It is not that we know things,[22] or believe in their existence, or assume them. It is true that, in a phrase reminiscent of Hegel, "we are compelled to think the object";[23] but this is only in the sense that things are perceived by the critical realist to the same degree and with the same actuality of details—though not in the same manner—as by the naïve realist. It is the essence of Critical Realism that, whatever be the true theory of perception, there is absolutely no difference whatever between what the critical realist actually perceives and what the naïve realist perceives—of course excluding, in both instances alike, ordinary error and illusion. The *result* in each case is precisely the same, for each perceives real physical things.[24]

[21] Pp. 196, 197.
[22] On p. 205 we find "I know an object." I consider this later.
[23] P. 198.
[24] Or objects; the distinction is immaterial here.

That is the essence of the claim to be perceptual realism; the only difference being that one understands how it occurs, while the other altogether misinterprets the true character of the processes involved.

It seems to me, however, that the two principles which in conjunction form the basis of Critical Realism—that is the subjectivity or internality of all perceptual content, and the physical nature (in the sense of naïve realism) of the things or objects perceived—can not logically be held at the same time; the maintenance of either necessarily involves the abandonment of the other. If all the physical qualities of the naïve realist are truly mere character complexes which constitute subjective content, then *physical things* can not be perceived, although they may possibly be known, or postulated, or believed in—this would depend on the meaning given to these terms. On the other hand, if physical things are perceived—again in the sense of naïve realism—then some concessions must be made to its standpoint as a serious theory of knowledge. Its "qualities" must be, to some degree at least, not merely representative or reproductive or reflective of the thing, but ontologically identical with it.

For if, as Critical Realism contends, *all* the content involved is, and always has been, subjective, then, although there certainly might have been realists of some type or another, there could never have been any naïve realists in the world at all; and since critical realism is confessedly a development from the primary naïve form, there could have been no critical realists either. In other words, those problems which actually confront epistemology would never have arisen. For those problems originate in the contrast which is set up by naïve realism between objective and subjective, externality and internality, matter and mind, or whatever other terms express this dichotomy. It rests upon certain deliverances of perceptive experience;

it can indeed have no other possible basis, since, as we have seen repeatedly, perception is accepted as foundational of knowledge. But if now the whole content of perception is subjective, it would have been impossible for this contrast, in its actual form, ever to have been in any way established, simply because there would have been nothing whatever in the universe of perception to suggest it to the experient—it would have had no perceptive basis actually, and it could not conceivably have acquired any other. It cannot be argued that it arises from the spatiality of the universe; for all the character traits of perceived space, like those of every other perceived object, are (by the critico-realist hypothesis) themselves subjective. Universal subjectivity of the basal elements of experience, in short, would render impossible (actually) and inconceivable (theoretically) that contrast between objective and subjective which has, as a matter of fact, actually occurred in the evolution of knowledge; for naïve realism, despite all its defects, is after all one marked stage in that evolution.[25]

This conclusion appears to me to vitiate the entire critico-realist standpoint; and further considerations may be advanced in its support. It may be argued that the mere practical demands of individual life and of mutual intercourse necessitate that category of physical objectivity which the realist—either naïvely or critically—employs; but I do not think this need necessarily be the case at all. It can make no difference, for low types of conscious experience, whether the "world" is all subjective or all objective; life would proceed equally well in either case, since neither category can have any significance for the experi-

[25] On account of its fundamental importance in epistemology I may be permitted to refer to a prior consideration of the subject, from a different point of view, in *Mind*, vol. xxvii, p. 304. *Cf.* "We mean independent objects and we interpret these objects in terms of ideas." (P. 194.) My contention may be put in the form that if these "independent objects" are physical, no notion of them could ever have arisen.

ent; while theoretically to regard all the content at that stage as subjective is a gratuitous assumption. In the opposite direction we find that when mind reaches its highest levels the mathematician, the abstract thinker, the poet and artist, perhaps even the true mystic, live in and efficiently react to a realm which wholly lacks physical objectivity. All that is theoretically necessary then for such efficient reaction is the selection—consciously or otherwise—of certain elements out of the whole which serve as types, quite irrespective of *physical* objectivity and subjectivity, and which other elements then represent or reflect. The symbols of the mathematician may be cited as one instance of this general procedure.

This naturally leads to the representative function which, as earlier quotations have shown, is assigned by critical realism to subjective perceptual content. Professor Drake accepts—but only provisionally—the well-known distinction between primary and secondary qualities; and this raises a dilemma already familiar in one form or another. The desk (pp. 23, 24), "really is oblong, but not in itself black, except in the sense that it has characteristics which cause the character-trait black to appear to us." If then the characteristics which cause the trait black to appear are themselves black, as (it would seem) other allied characteristics are oblong, where is the necessity for the trait at all? It is wholly gratuitous to lay down *a priori limits* to what lies within the capacity of the mind to perceive; and it seems sheer superfluity to require a black content whereby to perceive a black object whose blackness is similar to that of the content. But on the other hand, if the object's characteristic is not black, in the same way as the content is black, then what is it? Obviously, we can not tell; our ignorance is complete and final, for we can neither perceive nor conceive its quality or nature;

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which means that the thing is, to that degree, a noümenon and perception a pure misnomer.

But, as a matter of fact, critical realism does assert *a priori* limits to the capacity of perception. "What," asks Professor Sellars, "is the fundamental postulate of knowledge? It is the cognitive value of the idea."[26] But this principle, when thus advanced as the basis of a theory of perception, plainly commits realism in advance to complete dependence upon subjective ideas; it postulates an epistemology resting at bottom on ideas—in other words, it begs the main issue from the very outset. The sole fundamental postulate of epistemology is, not the cognitive value of ideas, but the cognitive value of knowledge itself. And this is not the mere tautology that, at first sight, it may appear to be; it means that knowledge, whatever be the precise character of its contributory processes, is always knowledge of reality. Some ideas most certainly have cognitive value; it may indeed prove to be true that nothing has cognitive value except ideas. But such a principle can be adopted only at the conclusion of the inquiry, as a logically established result; it can not be advanced at the outset, as a fundamental postulate. To do so is merely to base realism firstly on that radically defective sensationalistic tradition which vitiates so much current psychology, and which it should be the first task of realism to criticise and confute; and secondly on a misinterpretation—for which, however, some historic justification may be pleaded —of the true meaning of "idea."

In conclusion then it seems to me that realism must make substantial concessions to the naïve realist and—though not of course uncritically—adopt more generally his practical standpoint. At *some* levels of experience the content of perceived physical reality must be identical and consubstantial with the content of experience, and this

[26] P. 198.

without the mediation of any subjective or internal content of any type whatever in any kind of representative or reflective or reproductive capacity. It may easily prove to be the case that the volume of the content in question is, relatively to the whole, very small; but there is an infinity of difference between very little and none at all. We know that in the living organism a minute proportion of certain ions or of vitamines makes all the difference between life and death; and similarly some minimum of physical reality, perceived directly, not merely in the critico-realist sense, but independently of the critical realist's subjective and representative perceptual content, plays an analogous part in realistic epistemology.

A final remark on terminology may not be out of place here. Modes of consciousness vary correlatively with the content; and it seems to me to lead to some confusion if we speak of "knowing" a physical object or thing. Epistemology is, in its own way, a science; and quite apart from any theory of perception, I would suggest that scientific precision requires us to say that we "perceive" physical things; that is, the sole type of consciousness relevant to them is perception, with its subsidiary aspects of sensation and apperception. It will be admitted that we can neither imagine nor conceive existing physical objects, because these functions lack proper sensational content; and this is equally true of knowledge as such. What we "know" is a truth or principle, fact or judgment. Perception is certainly implicit judgment, and knowledge consists in rendering explicit what is there implicit. But still there is an essential difference between implicit and explicit; and to ignore this—to say that we "know" or "think" the object—appears to involve a serious confusion of perfectly distinct categories.

J. E. TURNER.

LIVERPOOL, ENGLAND.

EINSTEIN'S THEORY OF RELATIVITY CONSIDERED FROM THE EPISTEMOLOGICAL STANDPOINT*

VI. Euclidean and Non-Euclidean Geometry

IN THE preceding considerations, however, we have taken up only incidentally an achievement of the general theory of relativity, which, like scarcely a second, seems to involve a "revolution of thought." In the working out of the theory, it is seen that the previous Euclidean measurements are not sufficient; the development of the theory can only take place by our going from the Euclidean continuum, which was still taken as a basis by the special theory of relativity, to a non-Euclidean four-dimensional space-time continuum and seeking to express all relations of phenomena in it. Thus a question seems answered physically which had concerned the epistemology of the last decades most vitally and which had been answered most diversely within it. Physics now proves not only the possibility, but the reality of non-Euclidean geometry; it shows that we can only understand and represent theoretically the relations, which hold in "real" space, by reproducing them in the language of a four-dimensional non-Euclidean manifold.

The solution of this problem from the side of physics was, on the one hand, for a long time hoped for as keenly,

* Translated by W. C. and M. C. Swabey.

as, on the other hand, its possibility was vigorously denied. Even the first founders and representatives of the doctrine of non-Euclidean geometry sought to adduce experiment and concrete measurement in confirmation of their view. If we can establish, they inferred, by exact terrestrial or astronomical measurements, that in triangles with sides of very great length the sum of the angles differs from two right angles, then empirical proof would be gained that in "our" empirical space the propositions not of Euclidean geometry, but of one of the others were valid. Thus, *e. g.*, Lobatschefsky, as is known, used a triangle $E_1 E_2 S$, whose base $E_1 E_2$ was formed by the diameter of the orbit of the earth and whose apex S was formed by Sirius and believed that he could, in this way, prove empirically a possible constant curvature of our space. (48.) The fallacy in method of any such attempt must be obvious, however, to any sharper epistemological analysis of the problem and it has been pointed out from the side of the mathematicians with special emphasis by H. Poincaré. No measurement, as Poincaré objects with justice, is concerned with space itself but always only with the empirically given and physical objects in space. No experiment therefore can teach us anything about the *ideal* structures, about the straight line and the circle, that pure geometry takes as a basis; what it gives us is always only knowledge of the relations of material things and processes. The propositions of geometry are therefore neither to be confirmed nor refuted by experience. No experiment will ever come into conflict with the postulates of Euclid; but, on the other hand, no experiment will ever contradict the postulates of Lobatschefsky. For granted, that some experiment could show us a variation in the sums of the angles of certain very great triangles, then the conceptual representation of this fact would never need to consist in, and methodologi-

cally could not consist in, changing the axioms of geometry, but rather in changing certain hypotheses concerning physical things. What we would have *experienced,* in fact, would not be another structure of space, but a new law of optics, which would teach us that the propagation of light does not take place in strictly rectilinear fashion. "However, we turn and twist," Poincaré therefore concludes, "it is impossible to attach a rational meaning to empiricism in geometry." (72, p. 92ff.) If this decision holds and if it can be proved, on the other hand, that among all possible self-consistent geometries the Euclidean possesses a certain advantage of "simplicity" since it defines the minimum of those conditions under which experience is possible in general, there would then be established for it an exceptional position from the standpoint of the critique of knowledge. It would be seen that the different geometries, which are equivalent to each other from a purely formal standpoint, as regards their logical conceivability, are yet distinguished in their fruitfulness in the founding of empirical science. "The geometries are distinguished from each other in principle," one can conclude, "only by reference to their epistemological relation to the concept of experience; for this relation is positive only in the case of the Euclidean geometry."[29]

In connection, however, with the new development of physics in the general theory of relativity, this epistemological answer seems to become definitively untenable. Again and again the fact has been appealed to in the controversy concerning the epistemological justification of the different geometries that what determines value must not be sought in formal but in transcendental logic; that the compatibility of a geometry with experience is not involved but rather its "positive fruitfulness," *i. e.,* the " founding of

[29] *Cf.* Hönigswald (32); on the following *cf.* Bauch (1), p. 126ff.

experience," that it can give. And this latter was thought to be found in Euclidean geometry. The latter appeared as the real and unique "foundation of possibility of knowledge of reality," the others, on the contrary, always as only the foundations of the possible. But with regard to the extraordinary rôle that the concepts and propositions of Riemannian geometry played in the grounding and construction of Einstein's theory of gravitation, this judgment cannot be supported. Supported by the same logical criterion of value, one now seems forced rather to the opposite conclusion: non-Euclidean space is alone "real," while Euclidean space represents a mere abstract possibility. In any event, the logic of the exact sciences now finds itself placed before a new problem. The fact of the fruitfulness of non-Euclidean geometry for physics can no longer be contested, since it has been verified, not only in particular applications, but in the structure of a complete new system of physics; what is in question is the explanation to be given to this fact. And here we are first forced to a negative decision, which is demanded by the first principles of the theory of relativity. Whatever meaning we may ascribe to the idea of non-Euclidean geometry for physics, for purely empirical thought, the assertion has lost all meaning for us that any space, whether Euclidean or non-Euclidean, is the "real" space. Precisely this was the result of the general principle of relativity, that by it "the last remainder of physical objectivity" was to be taken from space. Only the various relations of measurement within the physical manifold, within that inseparable correlation of space, time, and the physically real object, which the theory of relativity takes as ultimate, are pointed out; and it is affirmed that these relations of measurement find their simplest exact mathematical expression in the language of non-Euclidean geometry. This language, how-

ever, is and remains purely ideal and symbolic, precisely as, rightly understood, the language of Euclidean geometry could alone be. The reality which alone it can express is not that of things, but that of laws and relations. And now we can ask, epistemologically, only one question: whether there can be established an exact relation and coördination between the symbols of non-Euclidean geometry and the empirical manifold of spatio-temporal "events." If physics answers this question affirmatively, then epistemology has no ground for answering it negatively. For the "a priori" of space that it affirms as the condition of every physical theory involves, as has been seen, no assertion concerning any definite particular structure of space in itself, but is concerned only with that function of "spatiality" in general, that is expressed even in the general concept of the linear element *ds* as such, quite without regard to its character in detail.

If it is seen thus, that the determination of this element as is done in Euclidean geometry, does not suffice for the mastery of certain problems of knowledge of nature then nothing can prevent us, from a methodological standpoint, from replacing it by another measure, in so far as the latter proves to be necessary and fruitful physically. But in either case one must guard against taking the "preestablished harmony between pure mathematics and physics," that is revealed to us in increasing fulness and depth in the progress of scientific knowledge, as a naïve copy theory. The structures of geometry, whether Euclidean or non-Euclidean, possess no immediate correlate in the world of *existence*. They exist as little physically in things as they do psychically in our "presentations" but all their "being," *i. e.*, their validity and truth, consists in their ideal *meaning*. The existence, that belongs to them by virtue of their definition, by virtue of a pure logical act of

assumption is, in principle, not to be interchanged with any sort of empirical "reality." Thus also the applicability, which we grant to any propositions of pure geometry, can never rest on any direct coinciding between the elements of the ideal geometrical manifold and those of the empirical manifold. In place of such a sensuous congruence we must substitute a more complex and more thoroughly mediate relational system. There can be no copy or correlate in the world of sensation and presentation for what the points, the straight lines and the planes of pure geometry signify. Indeed, we cannot in strictness speak of any degree of similarity, of greater or less difference of the "empirical" from the ideal, for the two belong to fundamentally different species. The theoretical relation, which science nevertheless establishes between the two, consists merely in the fact, that it, while granting and holding fast to the difference in content of the two series, seeks to establish a more exact and perfect correlation between them. All verification, which the propositions of geometry can find in physics, is possible only in this way. The particular geometrical truths or particular axioms, such as the principle of parallels, can never be compared with particular experiences, but we can always only compare with the whole of physical experience the whole of a definite system of axioms. What Kant says of the concepts of the understanding in general, that they only serve "to make letters out of phenomena so that we can read them as experiences" holds in particular of the concepts of space. They are only the letters, which we must make into words and propositions, if we would use them as expressions of the laws of experience. If the goal of harmony is not reached in this indirect way, if it appears that the physical laws to which observation and measurement lead us cannot be represented and expressed with sufficient exactitude and simplicity by a given system

of axioms, then we are free to determine which of the two factors we shall subject to a transformation to reëstablish the lost harmony between them. Before thought advances to a change of one of its "simple" geometrical laws it will first make the complex physical conditions that enter into the measurement responsible for the lack of agreement; it will change the "physical" factors before the "geometrical." If this does not lead to the goal and if it is seen, on the other hand, that surprising unity and systematic completeness can be reached in the formulation of the "laws of nature" by acceptng an altered conception of geometrical methods, then in principle there is nothing to prevent such a change. For if we conceive the geometrical axioms, not as copies of a given reality, but as purely ideal and constructive structures, then they are subjected to no other law than is given them by the *system* of thought and knowledge. If the latter proves to be realizable in a purer and more perfect form by our advancing from a relatively simpler geometrical system to a relatively more complex, then the criticism of knowledge can raise no objection from its standpoint. It will be obliged to affirm only this: that here too "no intelligible meaning can be gained" for empiricism in geometry. For here, too, experience does not *ground* the geometrical axioms, but it only makes from among them, as various logically possible systems, of which each one is derived strictly rationally, a certain selection as to their concrete use, as to the interpretation of phenomena.[30] Here, too, Platonically speaking, phenomena are measured by Ideas, by the foundations of geometry, and these latter are not directly read out of the sensuous phenomena.

But when one grants to non-Euclidean geometry in this sense meaning and fruitfulness for physical experience, the general methodic difference can and must be urged, that

[30] On this relation of the problem of metageometry to the problem of "experience," *cf.* esp. Albert Görland (28, p. 324ff.)

still remains between it and Euclidean geometry. This difference can no longer be taken from their relation to experience, but it must be recognized as based on certain "inner" moments, *i. e.*, on general considerations of the *theory of relations.* A special and exceptional logical position, a fundamental simplicity of ideal structure, can be recognized in Euclidean geometry even if it must abandon its previous sovereignty within physics. And here it is precisely the fundamental doctrine of the general theory of relativity, that, translated back from the language of logic and general methodology, can establish and render intelligible this special position. Euclidean geometry rests on a definite axiom of relativity, which is peculiar to it. As the geometry of space of a constant curvature O, it is characterized by the thorough-going relativity of all places and magnitudes. Its formal determinations are in principle independent of any absolute determinations of magnitude. While, *e. g.,* in the geometry of Lobatschefsky, the sum of the angles of a rectilinear triangle is different from 180° and indeed the more so, the more the surface area of the triangle increases, the absolute magnitude of the lines enters into none of the propositions of Euclidean geometry. Here for every given figure a "similar" can be constructed; the particular structures are grasped in their pure "quality," without any definite "quantum," any absolute value of number and magnitude, coming into consideration in their definition. This indifference of Euclidean structures to all absolute determinations of magnitude and the freedom resulting here of the particular points in Euclidean space of all determinations and properties, form a logically positive characteristic of the latter. For the proposition, *omnis determinatio est negatio,* holds here too. The assumption of the indeterminate serves as the foundation for the more complex assumptions and deter-

minations, that can join on to it. In this sense, Euclidean geometry is and remains the "simplest," not in any practical, but in a strictly logical meaning; Euclidean space is, as Poincaré expresses it, "simpler not merely in consequence of our mental habits or in consequence of any direct intuition, which we possess of it, but it is in itself simpler, just as a polynomial of the first degree is simpler than a polynomial of the second degree." (72, p. 67.) This logical simplicity belonging to Euclidean space in the system of our intellectual meanings wholly independently of its relations to experience, is shown, *e. g.*, in the fact that we can make any "given" space, that possesses any definite curvature, into Euclidean by regarding sufficiently small fields of it from which the difference conditioned by the curvature disappears. Euclidean geometry shows itself herein as the real geometry of infinitely small areas, and thus as the expression of certain elementary relations, which we take as a basis in thought, although we advance from them in certain cases to more complex forms.

The development of the general theory of relativity leaves this methodic advantage of Euclidean geometry unaffected. For Euclidean measurements do not indeed hold in it absolutely but they hold for certain "elementary" areas, which are distinguished by a certain simplicity of physical conditions. The Euclidean expression of the linear element shows itself to be unsatisfactory for the working out of the fundamental thought of the general theory of relativity, since it does not fulfill the fundamental demand of retaining its form in every arbitrary alteration of the system of reference. It must be replaced by the *general* linear element ($ds^2 = \sum_1^4 g_{\mu\nu} dx_\mu dx_\nu$), which satisfies this demand. If, however, we consider infinitely small four-dimensional fields, it is expressly demanded that the pre-

suppositions of the special theory of relativity, and thus its Euclidean measurements shall remain adequate for them. The form of the universal linear element here passes over into the Euclidean element of the special theory when the ten magnitudes g, which occur in this as functions of the coördinates of particular points assume definite constant values. The physical explanation of this relation, however, consists in that the magnitudes $g_{\mu\nu}$ are recognized as those which describe the gravitational field with reference to the chosen system of reference. The condition, under which we can pass from the presuppositions of the general theory of relativity to the special theory, can accordingly be expressed in the form that we only consider regions within which abstraction can be made from the effects of fields of gravitation. This is always possible for an infinitely small field and it holds further for finite fields in which, with appropriate choice of the system of reference, the body considered undergoes no noticeable acceleration. As we see, the variability of the magnitudes $g_{\mu\nu}$, which expresses the variation from the homogeneous Euclidean form of space, is recognized as based on a definite *physical* circumstance. If we consider fields in which this circumstance is absent or if we cancel it in thought, we again stand within the Euclidean world. Thus the assertion of Poincaré that all physical theory and physical measurement can prove absolutely nothing about the Euclidean or non-Euclidean character of *space,* since it is never concerned with the latter but only with the properties of *physical reality in space* remains thus entirely in force. The abstraction (or, better expressed, the pure function) of homogeneous Euclidean space is not destroyed by the theory of relativity, but is only known as such through it more sharply than before.

In fact, the pure meaning of geometrical concepts is not limited by what this theory teaches us about the conditions of measurement. These concepts are indeed, as is seen now anew, neither an empirical datum nor an empirical *dabile,* but their ideal certainty and meaning is not in the least affected thereby. It is shown that in fields where we have to reckon with gravitational effects of a definite magnitude, the preconditions of the ordinary methods of measurement fall aside, that here we can no longer use "rigid bodies" as measures of length, nor ordinary "clocks" as measures of time. But this change of relations of measurement does not affect the calculation of space, but the calculation of the physical relation between the measuring rods and rays of light determined by the field of gravitation. (*Cf.* 83, p. 85ff.) The truths of Euclidean geometry would only be also affected if one supposed that these propositions themselves are nothing but generalizations of empirical observation, which we have established in connection wth fixed bodies. Such a supposition, however, epistemologically regarded, would amount to a *petitio principii.* Even Helmholtz, who greatly emphasizes the empirical origin of the geometrical axioms occasionally refers to another view, which might save their purely ideal and "transcendental" character. The Euclidean concept of the straight line might be conceived not as a generalization from certain physical observations, but as a purely ideal concept, to be confirmed or refuted by no experience, since we would have to decide by it whether any bodies of nature were to be regarded as fixed bodies. But, as he objects, the geometrical axioms would then cease to be synthetical propositions in Kant's sense, as they would only affirm something that would follow analytically from the concepts of the fixed geometrical structures necessary to measurement. (30a, II, 30.) It is, however, overlooked by

this objection that there are *fundamentally synthetic forms of unity* besides the form of analytic identity, which Helmholtz has here in mind and which he contrasts with the empirical concept as if the form of analytic identity were unique, and that the axioms of geometry belong precisely to the former. Assumptions of this sort refer to the object in so far as in their totality they "constitute" the object and render possible knowledge of it; but none of them, taken for itself, can be understood as an assertion concerning things or relations of things. Whether they fulfill their task as moments of empirical knowledge can be decided always only in the indicated indirect way: by using them as building-stones in a theoretical and constructive system, and then comparing the consequences, which follow from the latter, with the results of observation and measurement. That the elements, to which we must ascribe, methodologically, a certain "simplicity," must be adequate for the interpretation of the laws of nature, can not be demanded *a priori*. But even so, thought does not simply give itself over passively to the mere *material* of experience, but it develops out of itself new and more complex *forms* to satisfy the demands of the empirical manifold.

If we retain this general view, then one of the strangest and, at first appearance, most objectionable results of the general theory of relativity receives a new light. It is a necessary consequence of this theory that in it one can no longer speak of an immutably given geometry of measurement, which holds once for all for the whole world. Since the relations of measurement of space are determined by the gravitational potential and since this is to be regarded as in general changeable from place to place, we cannot avoid the conclusion that there is in general no unitary "geometry" for the totality of space and reality, but that, according to the specific properties of the field of

gravitation at different places, there must be found different forms of geometrical structure. This seems, in fact, the greatest conceivable departure from the idealistic and Platonic conception of geometry, according to which it is the "science of the eternally existent," knowledge of what always "is in the same state" (ἀεὶ κατὰ ταὐτὰ ὡσαύτως ἔχον). Relativism seems here to pass over directly into the field of logic; the relativity of places involves that of geometrical truth. And yet this view is, on the other hand, only the sharpest expression of the fact that the problem of space has lost all ontological meaning in the theory of relativity. The purely methodological question has been substituted for the question of being. We are no longer concerned with what space "is" and with whether any definite character, whether Euclidean, Lobatschefskian or Riemannian, is to be ascribed to it, but rather with what use is to be made of the different systems of geometrical presuppositions in the interpretation of the phenomena of nature and their dependencies according to law. If we call any such system a particular "space," then indeed we can no longer attempt to grasp all of these spaces as intuitive parts to be united into an intuitive whole. But this impossibility rests fundamentally on the fact that we have here to do with a problem, which as such stands outside the limits of intuitive representation in general. The space of pure intuition is always only *ideal*, being only the space constructed according to the laws of this intuition, while here we are not concerned with such ideal syntheses and their unity, but with the relations of measurement of the empirical and the physical. These relations of measurement can only be gained on the basis of natural laws, *i. e.*, by proceeding from the dynamic dependency of phenomena upon each other, and by permitting phenomena to determine their positions reciprocally in the space-time mani-

fold by virtue of this dependency. Kant too decisively urged that this form of dynamic determination did not belong to intuition as such, but that it is the "rules of the understanding" which alone give the existence of phenomena synthetic unity and enable them to be collected into a definite concept of experience. (*Cf.* above, p. 79.) The step beyond him, that we have now to make on the basis of the results of the general theory of relativity, consists in the insight that geometrical axioms and laws of other than Euclidean form can enter into this determination of the understanding, in which the empirical and physical world arises for us, and that the admission of such axioms not only does not destroy the unity of the world, *i. e.*, the unity of our experiential concept of a total order of phenomena, but first truly grounds it from a new angle, since in this way the particular laws of nature, with which we have to calculate in space-time determination, are ultimately brought to the unity of a supreme principle,—that of the universal postulate of relativity. The renunciation of intuitive simplicity in the picture of the world thus contains the guarantee of its greater intellectual and systematic completeness. This advance, however, can not surprise us from the epistemological point of view; for it expresses only a general law of scientific and in particular of physical thought. Instead of speaking ontologically of the being or indeed of the coexistence of diversely constituted "spaces," which results in a tangible contradiction, the theory of relativity speaks purely methodologically of the possibility of necessity of applying different measurements, *i. e.*, different geometrical conceptual languages in the interpretation of certain physical manifolds. This possible application tells us nothing concerning the "existence" of spaces, but merely indicates that by an appropriate choice of geometrical presuppositions certain physical relations,

such as the field of gravitation or the electromagnetic field, can be described.

The connection between the purely conceptual thought, involved in the working out of the general doctrine of the manifold and order, and physical empiricism (*Empirie*) here receives a surprising confirmation. A doctrine, which originally grew up merely in the immanent progress of pure mathematical speculation, in the ideal transformation of the hypotheses that lie at the basis of geometry, now serves directly as the form into which the laws of nature are poured. The same functions, that were previously established as expressing the metrical properties of non-Euclidean space, give the equations of the field of gravitation. These equations thus do not need for their establishment the introduction of new unknown forces acting at a distance, but are derived from the determination and specialization of the general presuppositions of measurement. Instead of a new complex of things, the theory is satisfied here by the consideration of a new general complex of conditions. Riemann, in setting up his theory, referred to its future physical meaning, in prophetic words of which one is often reminded in the discussion of the general theory of relativity. In the "question as to the inner ground of the relations of measurement of space," he urges, "the remark can be applied that in a discrete manifold the principle of measurement is already contained in the concept of this manifold, but in the case of a continuous manifold it must come from elsewhere. Either the real lying at the basis of space must be a discrete manifold or the basis of measurement must be sought outside it in binding forces working upon it. The answer to this question can only be found by proceeding from the conception of phenomena, founded by Newton and hitherto verified by experience and gradually reshaping this by facts that cannot be explained

from it; investigations, which, like the one made here, proceed from universal concepts, can only serve to the effect that these works are not hindered by limitations of concepts and the progress in knowledge of the connection of things not hindered by traditional prejudices." (77.) What is here demanded is thus full freedom for the construction of geometrical concepts and hypotheses because only thereby can physical thought attain also full effectiveness, and face all future problems resulting from experience with an assured and systematically perfected instrument. But this connection is expressed, in the case of Riemann, in the language of Herbartian *realism*. At the basis of the pure form of geometrical space a real is to be found in which is to be sought the ultimate cause for the inner relations of measurement of this space. If we carry out, however, with reference to this formulation of the problem, the critical, "Copernican," revolution and thus conceive the question so that a real does not appear as a ground of space but so that space appears as an ideal ground in the construction and progress of knowledge of reality, there results for us at once a characteristic transformation. Instead of regarding "space" as a self-existent real, which must be explained and deduced from "binding forces" like other realities, we ask now rather whether the *a priori* function, the universal ideal relation, that we call "space" involves possible formulations and among them such as are proper to offer an exact and exhaustive account of certain physical relations, of certain "fields of force." The development of the general theory of relativity has answered this question in the affirmative; it has shown what appeared to Riemann as a geometrical hypothesis, as a mere possibility of thought, to be an organ for the knowledge of reality. The Newtonian dynamics is here resolved into pure kinematics and this kinematics ultimately into

geometry. The content of the latter must indeed by broadened and the "simple" Euclidean type of geometrical axioms must be replaced by a more complex type; but in compensation we advance a step further into the realm of being, *i. e.*, into the realm of empirical knowledge, without leaving the sphere of geometrical consideration. By abandoning the form of Euclidean space as an undivided whole and breaking it up analytically and by investigating the place of the particular axioms and their reciprocal dependence or independence, we are led to a system of pure *a priori* manifolds, whose laws thought lays down constructively, and in this construction we possess also the fundamental means for representing the relation of the real structures of the empirical manifold.

The realistic view that the relations of measurement of space must be grounded on certain physical determinations, on "binding forces" of matter, expresses this peculiar double relation one-sidedly and thus, epistemologically regarded, inexactly and unsatisfactorily. For this *metaphysical* use of the category of ground would destroy the *methodological* unity, which should be brought out. What relativistic physics, which has developed strictly and consistently from a theory of space and time measurement, offers us is in fact only the combination, the reciprocal determination, of the metrical and physical elements. In this, however, there is found no one-sided relation of ground and consequent, but rather a purely reciprocal relation, a correlation of the "ideal" and "real" moments, of "matter" and "form," of the geometrical and the physical. In so far as we assume any division at all in this reciprocal relation and take one element as "prior" and fundamental, the other as "later" and derivative, this distinction can be meant only in a logical, not in a real sense. In this sense, we must conceive the pure space-time manifold as the logi-

cal *prius;* not as if it existed and were given in some sense outside of and before the empirical and physical, but because it constitutes a principle and a fundamental condition of all knowledge of empirical and physical relations. The physicist as such need not reflect on this state of affairs; for in all the concrete measurements, which he makes, the spatio-temporal and the empirical manifold is given always only in the unitary operation of measurement itself, not in the abstract isolation of its particular conceptual elements and conditions.

From these considerations the relation between Euclidean and non-Euclidean geometry appears in a new light. The real superiority of Euclidean geometry seems at first glance to consist in its concrete and intuitive determinateness in the face of which all "pseudo-geometries" fade into logical "possibilities." These possibilities exist only for thought, not for "being"; they seem analytic plays with concepts, which can be left unconsidered when we are concerned with experience and with "nature," with the synthetic unity of objective knowledge. When we look back over our earlier considerations, this view must undergo a peculiar and paradoxical reversal. Pure Euclidean space stands, as is now seen, not closer to the demands of empirical and physical knowledge than the non-Euclidean manifolds but rather more removed. For precisely because it represents the logically simplest form of spatial construction it is not wholly adequate to the complexity of content and the material determinateness of the empirical. Its fundamental property of homogeneity, its axiom of the equivalence in principle of all points, now marks it as an abstract space; for, in the concrete and empirical manifold, there never is such uniformity, but rather thorough-going differentiation reigns in it. If we would create a conceptual expression for this fact of differentiation in the sphere of

geometrical relations themselves, then nothing remains but to develop further the geometrical conceptual language with reference to the problem of the "heterogeneous." We find this development in the construction of metageometry. When the concept of the special three-dimensional manifold with a curvature O is broadened here to the thought of a system of manifolds with different constant or variable curvatures, a new ideal means is discovered for the mastery of complex manifolds; new conceptual symbols are created, not as expressions of things, but of possible relations according to law. Whether these relations are realized within phenomena at any place only experience can decide. But it is not experience that grounds the content of the geometrical concepts; rather these concepts foreshadow it as methodological anticipations, just as the form of the ellipse was anticipated as a conic section long before it attained concrete application and significance in the courses of the planets. When they first appeared, the systems of non-Euclidean geometry seemed lacking in all empirical meaning, but there was expressed in them the intellectual preparation for problems and tasks, to which experience was to lead later. Since the "absolute differential calculus," which was grounded on purely mathematical considerations by Gauss, Riemann and Christoffel, gains a surprising application in Einstein's theory of gravitation, the possibility of such an application must be held open for all, even the most remote, constructions of pure mathematics and especially of non-Euclidean geometry. For it has always been shown in the history of mathematics that its complete freedom contains the guarantee and condition of its fruitfulness. Thought does not advance in the field of the concrete by dealing with the particular phenomena like pictures to be united into a single mosaic, but by sharpening and refining its own means of determination while

guided by reference to the empirical and by the postulate of its determinateness according to law. If a proof were needed for this logical state of affairs, the development of the theory of relativity would furnish it. It has been said of the special theory of relativity that it "substituted mathematical constructions for the apparently most tangible reality and resolved the latter into the former." (38, p. 13.) The advance to the general theory of relativity has brought this constructive feature of it more distinctly to light; but, at the same time, it has shown how precisely this resolution of the "tangible" realities has verified and established the connection of theory and experience in an entirely new way. The further physical thought advances and the higher universality of conception it reaches the more does it seem to lose sight of the immediate data, to which the naïve view of the world clings, so that finally there seems no return to these data. And yet the physicist abandons himself to these last and highest abstractions in the certainty and confidence of finding in them reality, *his* reality in a new and richer sense. In the progress of knowledge the deep words of Heraclitus hold that the way upward and the way downward are one and the same: ὁδὸς ἄνω κάτω υίη. Here, too, ascent and descent necessarily belong together: the direction of thought to the universal principles and grounds of knowledge finally proves not only compatible with the direction to the particularity of phenomena and facts, but its correlate and condition.

VII. The Theory of Relativity and the Problem of Reality

WE HAVE attempted to show how the new concept of nature and of the object, which the theory of relativity establishes, is grounded in the form of physical thought and only brings this form to a final conclusion and clarity. Physical thought strives to determine and to express in pure objectivity merely the natural object, but it thereby necessarily expresses itself, its own law and its own principle. Here is revealed again that 'anthropomorphism" of all our concepts of nature to which Goethe's wisdom of old age loved to point. "All philosphy of nature is still only anthropomorphism, *i. e.*, man, at unity with himself, imparts to everything that he is not, this unity, draws it into his unity, makes it one with him himself. . . . We can observe, measure, calculate, weigh, *etc.*, nature as much as we will, it is still only our measure and weight, as man is the measure of all things." Only, after all our preceding considerations, this "anthropomorphism" itself is not to be understood in a limited psychological way but in a universal, critical and transcendental sense. Planck points out, as the characteristic of the evolution of the system of theoretical physics, a progressive emancipation from anthropomorphic elements, which has as its goal the greatest possible separation of the system of physics from the individual personality of the physicist. (68, p. 7.) But into this "objective" system, free from all the accidents of individual standpoint and individual personality, there enter

those universal conditions of system, on which depends the peculiarity of the physical way of formulating problems. The sensuous immediacy and particularity of the particular perceptual qualities are excluded, but this exclusion is possible only through the concepts of space and time, number and magnitude. In them physics determines the most general content of reality, since they specify the direction of physical thought as such, as it were the form of the original physical apperception. In the formulation of the theory of relativity this reciprocal relation has been confirmed throughout. The principle of relativity has at once an objective and a subjective, or methodological meaning. The "postulate of the absolute world," which it involves according to an expression of Minkowski, is ultimately a postulate of absolute method. The general relativity of all places, times and measuring rods must be the last word of physics, because "relativization," the resolution of the natural object into pure relations of measurement constitutes the kernel of physical *procedure*, the fundamental cognitive function of physics.

If we understand, however, how, in this sense, the affirmation of relativity develops with inner consequence and necessity out of the very form of physics, a certain critical limitation of this affirmation also appears. The postulate of relativity may be the purest, most universal and sharpest expression of the physical concept of objectivity, but this concept of the *physical* object does not coincide, from the standpoint of the general criticism of knowledge, with reality absolutely. The progress of epistemological analysis is shown in that the assumption of the simplicity and oneness of the concepts of reality is recognized more and more as an illusion. Each of the original directions of knowledge, each interpretation, which it makes of phenomena to combine them into the unity of a theoretical

connection or into a definite unity of meaning, involves a special understanding and formulation of the concept of reality. There result here not only the characteristic differences of meaning in the objects of science, the distinction of the "mathematical" object from the "physical" object, the "physical" from the "chemical," the "chemical" from the biological," but there occur also, over against the whole of *theoretical* scientific knowledge, other forms and meanings of independent type and laws, such as the ethical, the aesthetic "form." It appears as the task of a truly universal criticism of knowledge not to level this manifold, this wealth and variety of forms of knowledge and understanding of the world and compress them into a purely abstract unity, but to leave them standing as such. Only when we resist the temptation to compress the totality of forms, which here result, into an *ultimate* metaphysical unity, into the unity and simplicity of an absolute "world ground" and to deduce it from the latter, do we grasp its true concrete import and fullness. No individual form can indeed claim to grasp absolute "reality" as such and to give it complete and adequate expression. Rather if the thought of such an ultimate definite reality is conceivable at all, it is so only as an Idea, as the problem of a totality of determination in which each particular function of knowledge and consciousness must coöperate according to its character and within its definite limits. If one holds fast to this general view, there results even within the pure concepts of nature a possible diversity of approaches of which each one can lay claim to a certain right and characteristic validity. The "nature" of Goethe is not the same as that of Newton, because there prevail, in the original *shaping* of the two, different principles of form, types of synthesis, of the spiritual and intellectual combination of the phenomena. Where there exist such diversities in fun-

damental *direction* of consideration, the *results* of consideration cannot be directly compared and measured with each other. The naïve realism of the ordinary view of the world, like the realism of dogmatic metaphysics, falls into this error, ever again. It separates out of the totality of possible concepts of reality a single one and sets it up as a norm and pattern for all the others. Thus certain necessary formal points of view, from which we seek to judge and understand the world of phenomena, are made into things, into absolute beings. Whether we characterize this ultimate being as "matter" or "life," "nature" or "history," there always results for us in the end confusion in our view of the world, because certain spiritual functions, that coöperate in its construction, are excluded and others are over-emphasized.

It is the task of systematic philosophy, which extends far beyond the theory of knowledge, to free the idea of the world from this one-sidedness. It has to grasp the *whole system* of symbolic forms, the application of which produces for us the concept of an ordered reality, and by virtue of which subject and object, ego and world are separated and opposed to each other in definite form, and it must refer each individual in this totality to its fixed place. If we assume this problem solved, then the rights would be assured, and the limits fixed, of each of the particular forms of the concept and of knowledge as well as of the general forms of the theoretical, ethical, aesthetic and religious understanding of the world. Each particular form would be "relativized" with regard to the others, but since this "relativization" is throughout reciprocal and since no single form but only the systematic totality can serve as the expression of "truth" and "reality," the limit that results appears as a thoroughly immanent limit, as one that is

removed as soon as we again relate the individual to the system of the whole.

We trace the general problem, which opens up here, no further but use it merely to designate the limits, that belong to any, even the most universal, *physical* formulation of problems, because these limits are necessarily grounded in the concept and essence of this way of formulating the question. All physics considers phenomena under the standpoint and presupposition of their measurability. It seeks to resolve the structure of being and process ultimately into a pure structure or order of numbers. The theory of relativity has brought this fundamental tendency of physical thought to its sharpest expression. According to it the procedure of every physical "explanation" of natural process consists in coördinating, to each point of the space-time continuum, four numbers, x_1, x_2, x_3, x_4, which possess absolutely no direct physical meaning but only serve to enumerate the points of the continuum "in a definite, but arbitrary way." (18, p. 64.) The ideal, with which scientific physics began with Pythagoras and the Pythagoreans, finds here its conclusion; all qualities, including those of pure space and time, are translated into pure numerical values. The logical postulate contained in the concept of number, which gives this concept its characteristic form, seems now fulfilled in a degree not to be surpassed; all sensuous and intuitive heterogeneity has passed into pure homogeneity. The classical mechanics and physics seeks to reach this immanent goal of conceptual construction by relating the manifold of the sensuously given to the homogeneous and absolutely uniform time. All difference of sensation is hereby reduced to a difference of motions; all possible variety of content is resolved into a mere variety of spatial and temporal positions. But the ideal of strict homogeneity is not reached here

since there are still always two fundamental forms of the homogeneous itself that are opposed to each other as pure space and pure time. The theory of relativity in its development advances beyond this opposition also; it seeks to resolve not only the differences of sensation but also those between spatial and temporal determinations into the unity of numerical determinations. The particularity of each "event" is expressed by the four numbers x_1, x_2, x_3, x_4, whereby these numbers among themselves have reference to no inner differences, so that some of them x_1, x_2, x_3, cannot be brought into a special group of "spatial" coördinates and contrasted with the time coördinate" x_4. Thus all differences belonging to spatial and temporal apprehension in subjective consciousness seem to be consistently set aside in the same way that nothing of the subjective visual sensation enters into the physical concept of light and color.[31] Not only are all spatial and temporal values exchangeable with each other, but all inner differences of the temporal itself, unavoidable for the subjective consciousness, all differences of *direction*, which we designate by the words "past" and "future," are cancelled. The direction into the past and that into the future are distinguished from each other in this form of the concept of the world by nothing more than are the + and — directions in space, which we can determine by arbitrary definition. There remains only the "absolute world" of Minkowski; the world of physics changes from a *process* in three-dimensional world in which time is replaced as a variable magnitude by the imaginary "ray of light" (*Lichtweg*) ($x_4 = \sqrt{-1}\, c\, t$).[32]

This transformation of the time-value into an imaginary numerical value seems to annihilate all "reality" and qualitative determinateness, which time possesses as the

[31] On this latter point *cf.* now Planck, *Das Wesen des Lichts* (71).
[32] *Cf.* Minkowski (54, p. 62ff.); Einstein (18, p. 82f.).

"form of the inner sense," as the form of immediate experience. The "stream of process," which, psychologically, constitutes consciousness and distinguishes it as such, stands still; it has passed into the absolute rigidity of a mathematical cosmic formula. There remains in this formula nothing of that form of time, which belongs to all our experience as such and enters as an inseparable and necessary factor into all its content.[33] But, paradoxical as this result seems from the standpoint of this experience, it expresses only the course of mathematical and physical objectification, for, to estimate it correctly from the epistemological standpoint, we must understand it not in its mere result, but as a process, a method. In the resolution of subjectively experienced qualites into pure objective numerical determinations, mathematical physics is bound to no fixed limit. It must go its way to the end; it can stop before no form of consciousness no matter how original and fundamental; for it is precisely its specific cognitive task to translate everything enumerable into pure number, all quality into quantity, all particular forms into a universal order and it only "conceives" them scientifically by virtue of this transformation. Philosophy would seek in vain to bid this tendency halt at any point and to declare *ne plus ultra.* The task of philosophy must rather be limited to conceiving this *meaning* in its logical dependency by recognizing fully the logical *meaning* of the mathematical and physical concept of objectivity. All particular physical theories including the theory of relativity receive their definite meaning and import only through the unitary cognitive will of physics, which stands back of them. The moment that we transcend the field of physics and change not the means but the very goal of knowledge, all particular concepts assume a new aspect and form. Each of these concepts means something different, depending on the general "modality"

[33] *Cf., e. g.,* J. Cohn (14, p. 228ff.).

of consciousness and knowledge with which it stands and from which it is considered. Myth and scientific knowledge, the logical and the aesthetic consciousness, are examples of such diverse modalities. Occasionally concepts of the same name, but by no means of the same meaning, meet us in these different fields. The conceptual relation, which we generally call "cause" and "effect" is not lacking to mythical thought, but here its meaning is specifically distinct from the meaning that it receives in scientific, and in particular, in mathematical and physical thought. In a similar way, all the fundamental concepts undergo a characteristic intellectual change of meaning when we trace them through the different fields of intellectual consideration. Where the copy theory of knowledge seeks a simple identity, the functional theory of knowledge sees complete diversity, but, indeed, at the same time complete correlation of the individual forms.[34]

If we apply these considerations to the concepts of space and time, then it is obvious what the transformation of these concepts in modern physics means, in its philosophical import, and what it cannot mean. The content of physical deductions cannot, without falling into the logical error of a μετάβασις εἰς ἄλλο γένος be simply carried over into the language of fields whose structure rests on a totally different structural principle. Thus, what space and time are as immediate contents of experience and as they offer themselves to our psychological and phenomenological analysis is unaffected by the use we make of them in the determination of the object, in the course of objective conceptual knowledge. The distance between these two types of consideration and conception is only augmented by the theory of relativity and thus only made known more distinctly, but is not first produced by it. Rather it is clear

[34] I am aware of the fragmentary character of these suggestions; for their supplementation and more exact proof I must refer to some subsequent more exhaustive treatment. *Cf.* also the essay *Goethe und die mathematische Physik* (11).

that even to attain the first elements of mathematical and physical knowledge and of the mathematical and physical object we assume that characteristic transformation of "subjective" phenomenal space and of "subjective" phenomenal time, which leads, in its ultimate consequences, to the results of the general theory of relativity. From the standpoint of strict sensualism too, it is customary to admit this transformation, this opposition between the "physiological" space of our sensation and presentation and the purely "metrical" space, which we make the basis of geometry. The latter rests on the assumption of the equivalence of all places and directions, while for the former the distinction of places and directions and the marking out of one above the others is essential. The space of touch, like that of vision, is anistropic and inhomogeneous, while metrical Euclidean space is distinguished by the postulate of isotropism and homogeneity. Compared with "metrical" time, physiological time shows the same characteristic variations and differences of meaning; one must, as Mach himself urges, as clearly distinguish between the immediate sensation of duration and the measuring number as between the sensation of warmth and temperature.[35]

[35] Mach (50, p. 331ff., 415ff.). If, with Schlick (79, p. 51ff.), one would call the psychological space of sensation and presentation the space of intuition, and contrast with it physical space as a conceptual construction, no objection could be made against this as a purely terminological determination; but one must guard against confusing this use of the word "intuition" with the Kantian, which rests on entirely different presuppositions. When Schlick sees in the insight that objective physical time has just as little to do with the intuitive experience of duration as the three-dimensional order of objective space with optical or "haptical" extension, "the kernel of truth in the Kantian doctrine of the subjectivity of time and space," and when he, on the other hand combats, on the basis of this distinction, the Kantian concept of "pure intuition," this rests on a psychological misunderstanding of the meaning of the Kantian concepts. The space and time of pure intuition are for Kant never sensed or perceived space or time, but the "mathematical" space and time of Newton; they are themselves constructively generated, just as they form the presupposition and foundation of all further mathematical and physical construction. In Kant's thought, "pure intuition" plays the rôle of a definite fundamental *method of objectification;* it coincides in no way with "subjective," *i. e.*, psychologically experienceable time and space. When Kant speaks of the subjectivity of space and time, we must never understand experiential subjectivity but their "transcendental" subjectivity as conditions of the possibility of "objective," *i. e.*, of objectifying empirical knowledge. (*Cf.* also the significant remarks of Selliens against Schlick; 81, p. 19, 39.)

This contrast between subjective, "phenomenal" space and time, on the one hand, and objective and mathematical space and time, on the other, comes to light with special distinctness, when one considers a property which seems at first glance to be common to them. Of both we are accustomed to predicate the property of *continuity,* but we understand thereby, more closely considered, in the two cases something wholly different. The continuity, which we ascribe to time and processes in it on the basis of the form of our experience, and that which we define in mathematical concepts by certain constructive methods of analysis, not only do not coincide but they differ in their essential moments and conditions. The experiential continuity affirms that each temporal content is given to us only in the way of certain characteristic *"wholes,"* which can not be resolved into ultimate simple *"elements";* analytic continuity demands reduction to such elements. The first takes time and duration as "organic" unities in which according to the Aristotelian definition, "the whole precedes the parts"; the second sees in them only an infinite *totality* of parts, of particular sharply differentiated *temporal points.* In the one case, the continuity of becoming signifies that living flux, that is given to our consciousness only as a flux, as a transition, but not as separated and broken up into discrete parts; in the other, it is demanded that we continue our analysis beyond all limits of empirical apprehension; it is demanded that we do not allow the division of elements to cease where sensuous perception, which is bound to definite but accidental limits in its capacity for discrimination, allows it to end, but that we follow it purely intellectually *ad infinitum.* What the mathematician calls the "continuum" is thus never the purely experiential quality of "continuity," of which there is no longer possible any further "objective" definition, but it is a purely conceptual construction, which he puts in the place of the latter. Here

too he must follow his universal method; he must reduce the quality of continuity to mere number, *i. e.*, precisely to the fundamental form of all intellectual *discreteness*. (Cf. 6, p. 21.) The only continuum he knows and the one to which he reduces all others, is always the continuum of *real numbers* which modern analysis and theory of groups seek, as is known, to construct strictly conceptually with renunciation in principle of any appeal to the "intuition" of space and time. The continuum thus considered, as Henri Poincaré especially has urged with all emphasis is nothing but a totality of individuals, which are conceived in a definite order and are given indeed in infinite number, of which each one is opposed to the others as something separate and external. We are here no longer concerned with the ordinary view, according to which there exists between the elements a sort of "inner bond" by which they are connected into a whole, so that, *e. g.*, the point does not precede the line, but the line the point. "Of the famous formula, that the continuum is the unity of the manifold," concludes Poincaré, "there remains only the manifold,—the unity has disappeared. The analysts are nevertheless right when they define continuity as they do, for in all their inferences they are concerned, in so far as they claim rigor, only with this concept of the continuous. But this circumstance suffices to make us attentive to the fact that the true mathematical continuum is something totally different from that of the physicist and the metaphysician." (72, p. 30.) In so far as physics is an objectifying science working with the conceptual instruments of mathematics, the physical continuum is conceived by it as related to and exactly correlated with the mathematical continuum of pure numbers. But the "metaphysical" continuum of the pure and original "subjective" form of experience can never be comprehended in this way, for the very *direction* of mathematical consideration is such that, instead of leading to this

form, it continually leads away from it. The critical theory of knowledge, which does not have to select from among the different sorts of knowledge, but merely to establish what each of them "is" and means, can make no normative decision as to the opposite aspects under which the continuum here appears, but its tasks consists in defining the two with reference to each other in utmost distinctness and clarity. Only by such a delimitation can be reached, on the one hand, the goal of phenomenological analysis of the temporal and spatial consciousness, and on the other hand, the goal of the exact foundation of mathematical analysis and its concepts of space and time. "With regard to the objection," a modern mathematical author concludes his investigation of the continuum, "that nothing is contained in the intuition of the continuum of the logical principles that we must adduce in the exact definition of the concept of the real number, we have taken account of the fact that what can be found in the intuitive continuum and in the mathematical world of concepts are so alien to each other, that the demand that the two coincide must be rejected as absurd. In spite of this, those abstract schemata, which mathematics offers us, are helpful in rendering possible an exact science of fields of objects in which continua play a rôle. The exact temporal or spatial point does not lie in the given (phenomenal) duration or extension as an ultimate indivisible element, but only reason reaching through this can grasp these ideas and they crystallize into full determinateness only in connection with the purely formal arithmetical and analytical concept of the real number." [36]

If we bear in mind this state of affairs, the deductions of the theory of relativity in its determination of the four dimensional space and time continuum lose the appearance of paradox, for it is seen that they are only the final conse-

[36] Weyl, 84, p. 83, 71.

quence and working out of the fundamental methodic idea on which rests mathematical analysis in general. But the question as to which of the two forms of space and time, the psychological or the physical, the space and time of immediate experience or of mediate conception and knowledge, expresses the *true* reality has lost fundamentally for us all definite meaning. In the complex that we call our "world," that we call the being of our ego and of things, the two enter as equally unavoidable and necessary moments. We can cancel neither of them in favor of the other and exclude it from this complex, but we can refer each to its definite *place* in the whole. If the physicist, whose problem consists in objectification, affirms the superiority of "objective" space and time over "subjective" space and time; if the psychologist and the metaphysician, who are directed upon the totality and immediacy of experience draw the opposite conclusion; then the two judgments express only a false "absolutization" of the norm of knowledge by which each of them determines and measures "reality." In which direction this "absolutization" takes place and whether it is directed on the "outer" or the "inner" is a matter of indifference to the purely epistemological judgment. For Newton it was certain that the absolute and mathematical time, which by its nature flowed uniformly, was the "true" time of which all empirically given temporal determination can offer us only a more or less imperfect copy; for Bergson, this "true" time of Newton is a conceptual fiction and abstraction, a barrier, which intervenes between our apprehension and the original meaning and import of reality. But it is forgotten that what is here called absolute reality, *dureé rèelle,* is itself no absolute but only signifies a standpoint of consciousness opposed to that of mathematics and physics. In the one case, we seek to gain a unitary and exact measure for all objective process, in the other we are concerned in retain-

ing this process itself in its pure qualitative character, in its concrete fulness and subjective inwardness and "contentuality." The two standpoints can be understood in their meaning and necessity; neither suffices to include the actual whole of being in the idealistic sense of "being for us." The symbols that the mathematician and physicist take as a basis in their view of the external and the psychologist in his view of the inner, must both be understood as *symbols.* Until this has come about, the true philosophical view, the view of the *whole,* is not reached, but a partial experience is hypostasized into the whole. From the standpoint of mathematical physics, the total content of the immediate qualities, not only the differences of sensation, but those of spatial and temporal consciousness, is threatened with complete annihilation; for the metaphysical psychologist, conversely, all reality is reduced to this immediacy, while every mediate conceptual cognition is given only the value of an arbitrary convention produced for the purposes of our action. But both views prove, in their absoluteness, rather perversions of the full import of being, *i. e.*, of the full import of the *forms* of knowledge of the self and the world. While the mathematician and the mathematical physicist stand in danger of permitting the real world to be identified with the world of their *measures,* the metaphysical view, in seeking to narrow mathematics to practical goals, loses the sense of its purest and deepest *ideal* import. It violently closes the door against what, according to Plato, constitutes the real meaning and the real value of mathematics; that, namely, "by each of these cognitions an *organ of the soul* is purified and strengthened, which under other occupations is lost and blinded; for its preservation is more important than that of a thousand eyes: for by this alone is the truth seen." And been the two poles of consideration, which we find here, there stand the manifold concepts of truth of the different concrete sciences—

and therewith their concepts of space and time. *History,* to set up its temporal measure, cannot do without the methods of the objectifying sciences: chronology is founded on astronomy and through this on mathematics. But the time of the historian is nevertheless not identical with that of the mathematician and physicist, but possesses in contrast to it a peculiar concrete form. In the concept of time of history, the "objective" content of knowledge and the "subjective" experiential content enter into a new characteristic reciprocal relation. An analogous relation is presented, when we survey the aesthetic meaning and shaping of the forms of space and time. Painting presupposes the objective laws of perspective, architecture the laws of statics, but the two serve here only as material out of which develops the unity of the picture and of the architectural spatial form, on the basis of the original artistic laws of form. For music, too, the Pythagoreans sought a connection with pure mathematics, with pure number; but the unity and rythmical division of a melody rests on wholly different structural principles than those on which we construct time in the sense of the unity of objective physical processes of nature. What space and time truly *are* in the philosophical sense would be determined if we succeeded in surveying completely this wealth of nuances of intellectual meaning and in assuring ourselves of the underlying formal law under which they stand and which they obey. The theory of relativity cannot claim to bring this philosophical problem to its solution; for, by its evolution and scientific tendency from the beginning, it is limited to a definite particular motive of the concepts of space and time. As a physical theory it merely develops the meaning that space and time possess in the system of our empirical and physical measurements. In this sense, final judgment on it belongs exclusively to physics. In the course of its history, physics will have to decide whether the world- picture of

the theory of relativity is securely founded theoretically and whether it finds complete experimental verification. Its decision on this, epistemology cannot anticipate; but even now it can thankfully receive the new incitements which this theory has given the general doctrine of the principles of physics.

AUTHOR'S NOTE

The above essay, of which this is the concluding section, does not claim to give a complete account of the philosophical problems raised by the theory of relativity. I am aware that the new problems presented to the general criticism of knowledge by this theory can only be mastered by the gradual and common work of physicists and philosophers; here I was merely concerned with beginning this work, with stimulating discussion, and, where possible, guiding it into definite methodic paths, in contrast to the uncertainty of judgment which still reigns. The purpose of this writing would be attained if it succeeded in preparing for a mutual understanding between the philosopher and the physicist on questions, concerning which they are still widely separated. That I was concerned, in purely epistemological matters, also, to hold myself in closest contact with scientific physics and that the writings of the leading physicists of the past and present have everywhere essentially helped to determine the intellectual orientation of the preceding investigation, will be gathered from the exposition. The bibliography, which follows, however, makes no claim to actual completeness; in it only such works are adduced as have been repeatedly referred to and intensively considered in the course of the exposition.

Albert Einstein read the above essay in manuscript and improved it by his critical comments; I cannot let it go out without expressing here also my hearty thanks to him.

ERNST CASSIRER.

UNIVERSITY OF HAMBURG.

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THE PSYCHO-GENESIS OF SPACE

DE MORGAN declares in the introduction to his *Budget of Paradoxes* that the sense or nonsense displayed by a paradoxer "will not depend upon what he maintains, but upon whether he has or has not made a sufficient knowledge of what has been done by others, *especially as to the mode of doing it,* a preliminary to inventing knowledge for himself." We may well accept this test of a veteran student of paradoxes to guide us aright in the face of the flood of paradoxes let loose in the name of "relativism" by a hitherto sober physical science. If we take a paradox in De Morgan's sense as "something which is apart from general opinion," then we are indeed confronted by the master paradox of all in the treatment of time as a fourth dimension. There is plenty of warrant for the suspicion that Minkowski, largely responsible for this development, has merely uttered the paradoxes of Zeno over again with the important difference that he believes them, whereas Zeno did not. Zeno employed his paradoxes to show that motion is inconceivable in terms of added states of rest; whereas Minkowski seeks to show that not only motion but the time it involves may be treated as a dimension. It would be difficult to invent a paradox that would so completely contradict the daily experiences of men. Yet, we must remember Copernicus, and with him the lesson that paradoxes may become commonplaces. It is the purpose of this paper to consider the grounds upon

which the paradox of Minkowski may look forward to such a happy outcome.

We needed to realize at the outset that the issue is a far more significant one than we ordinarily take paradoxes to be. We shall see that here is a conflict that can be traced back to that uncertain dawn when man first sought for the meaning of his experiences. Throughout the history of philosophy, where the external conflicts recorded by the historian, are seen as internal conflicts in the souls of men, the strife between two opposed conceptions of time are never abated. The *time of the clock* and *time lived* form the central issues in the conflict between mechanism and vitalism in biology, between continuity and relativity in physics, and between intellectualism and humanism in philosophy.

What are these opposed conceptions of time? Consider the wholly opposed nature of time *as lived* and time *as measured* in the simple experience of drinking a cup of tea. If thirsty, we impatiently wait for the lump of sugar to dissolve, and the time it requires *seems* to us far greater than the *actual* time recorded by the placid clock on the wall. When, however, the lump is finally dissolved, the time *seems* to pass quickly with the pleasure of drinking; but the clock is placid as before. Curiously enough, however, when these events are recalled later on, it will appear that the interval during which we waited will seem short and the succeeding interval of drinking will seem long. Hitherto, in deference to the clock, we have considered its time as *real* and our own time as *illusory*. It will be the purpose of this paper to ask if the very reverse is not the truth, namely: that the time lived is *absolute* whereas that of the clock is only *relative*. Second, it will be my aim to show that the paradox of Minkowski has grown out of the error of supposing that relative time, the time of the clock, is the only time there is.

Time as lived, far from resembling the divisions of a scale, cannot be represented by a line or any conceivable arrangement of them. Time as lived does not consist of separate moments in juxtaposition, but is a uniform inter-penetration. The time of the clock is reversible: for the purposes of the laboratory it is immaterial whether the clock runs backwards or forwards; and the fact that it repeats itself after every twenty-four hours is a great convenience. An astronomer, observing the transit of a star, may well turn back the hands of his clock or set his pendulum swinging anew, but the time he has wasted in wrong computations cannot be recovered so easily. Time as lived is irreversible, and in just this does the stinging quality of its reality consist.

In truth, there is no spatial figure that can possibly represent time in its fullness; and we can now see that the above differences are only the consequences of a more deep-seated difference between them. Eventually, the fundamental difference between the two conceptions inheres in the circumstance that in mathematical time it is assumed that creation is impossible whereas in the opposed conception it is assumed that creation is unavoidable. It is because science holds that the events with which it deals do not profit by their experiences that it is able to hold to its doctrine of conservation. The power of science to predict rests in its turn on the doctrine of conservation. In short, prediction in science proceeds by eliminating psychological time with the train of the unforeseen that it drags into all calculation. Mechanics is a mathematical science in which prediction is so perfect that time may be considered as non-existent. We can predict the "future" of a machine because it does not "learn" no matter how frequent its repetitions. Conversely, the very fact that a machine and a machine alone is capable of perfect repetition is because it is in capable of improving by practice. On the other hand,

a living organism cannot help but improve by practice, and this is because the past is conserved and to it is added the increased power brought by every new experience. It is clear that we might equally well have considered our own time as *real* and that of the clock as *appearance,* for who knows at first hand but that the rate of physical changes go now slow and now fast, leaving the *real* time within us pleased or impatient as the case may be? It thus appears that the problem is first a problem of definition; and it must be said that not a little of the disputation that the problem has exacted in contemporary speculation is the outgrowth of assigned mixed meanings to the terms involved.

What term is properly applicable to the experience of time as lived? This is the sole problem facing us. I believe we prepare the way for insoluble difficulties if instead of this simple ideal we suppose ourselves gauging the truth or falsity of a view from their harmony, not with our experience, but with some supposed "truth" underlying it. We shall not say that what we are seeking is knowledge that will correspond with "reality" unless by reality we mean no more than our experiences. The objection to the term *reality* is that it viciously conceals assumptions that in the metaphysical thinking of the past has given rise to endless trouble. Idealism has taken the term to refer to a universe of truth supposed to subsist without relation to human knowledge. Thus, they are able to save the possibility of knowledge only by the assumption that this reality is really known, and it forthwith declares that reality to be in the nature of an eternally complete and ideal universe. From this position, aside from the flat contradiction it contains, it will be forever impossible to comprehend the nature of human experience, which now becomes the unknown. Of matter, which is not ideal, and finite events, which are certainly not "complete," absolute idealism can give only two accounts. It may call pyramids

unreal or "appearances" because they are not *per se* ideas, but if this is taken as it is intended, to mean their non-being it is absurd. Ontologically speaking, "appearances" are just as real as anything we can know, and seem some how to get themselves talked about much more than the eternal verities that are supposed to expose their ghostly nature. On the other hand, if it is meant that pyramids are less real than their eternally ideal counterparts we insist that the proper terms *actual* or *potential* be employed. Materialism, on the other hand, believing as Flint says that "the seen is more potent than the unseen" only rivals the jargon of idealism by its profitless distinction between a *real* phenomenal universe and an *unreal* or epiphenomenal one. The term *unreal* has here been used to mean at once a dual implication of *non-existence* and *insignificance*.

What idealism and materialism plainly do is to demand that experience conform to preconceived ideas about it. This leaves all discourse the egoistic preoccupation of each man proving his case by denying all negative evidence; and history has shown that in such situations the supposed custodians of truth prefer to convince the heads of others by breaking them. Thought, instead of solving problems misses its function by denying their existence. Conceptions indeed form the substance of knowledge, but it is the business of discourse, not to force experience into conceptions already given but to find such conceptions in proper number, as will represent experience in its true unity or diversity.

I shall now point out how the above considerations will concern us in discussing space and time as forms of experience. It is a mistake to suppose that even were time proved to be "relative" that it therefore becomes unreal in any sense. What relativists must mean when they call time "unreal" is *that time conceived as an absolute* is unreal, i. e., not verified by experience. If time is truly so

relative that all its characteristics can be represented by a dimension then it is in every sense as real as any or all of the other three. In fine, the issue is solely to decide whether the experience of time can in all respects be considered relative or whether it has characteristics that demand the opposite category, the *absolute*. We may next inquire into the necessary meanings of these latter terms.

By the term relative we mean to indicate the dependence of the signification of anything by its comparison with something else. In order to have such a comparison at all it must be supposed that the terms of the comparison are alike in kind. In short, their comparison can only be in *quantitative* terms. The term *relative,* then, is of necessity inapplicable to the *qualitative* aspect of experience. It may be said that qualities are surely comparable in terms of their *intensity,* but reflection will show that in every idea of intensity there is hidden at once the notion of real and of implied space: it illegimately conveys two contradictory ideas, the idea of an actual space and the idea of a potential one. Space itself is none other than the field where *relations* are found: this is its actual service to science. It is misleading to say that space is the sum total of all points for points *per se* may be anywhere and actually are nowhere until they are given a locus by a relation. Space is a field of "anyness" rendered concrete by a relation, but science rightly holds that relations when empirically established are more than the "mental constructs" of Kant. The substance of the relation consists of the distance it measures between points. This distance, to be sure, may be construed to mean either the sum total of the unit distances comprising it or the effort that is required to traverse it. In exact science distances are always understood in the first sense, and are expressed by numbers. Thus, there is saved from the meaure taken, only its quantitative aspect—unless we inquire too closely into the inner

meaning that numbers have for us. So long, however, as numbers are regarded as symbols of pure quantity, those quantities may be decreased or increased at will so long as these changes take place proportionally, hence the accuracy of maps. As geographers, the difference in size between the map and the area it represents are very nearly negligible—not quite negligible, because however minute the map its various points still have to be traversed and hence it will still consume time, though far less time than it would take to traverse the surface it represents. As mathematicians, however, we reach the apex of all possible achievement in map-making for now distances may be represented by figures and any figure may represent any distance desired. In short, we have sacrificed all quality for the sake of schematism, but the sacrifice is only temporary for the reason that scheme making is not the whole of life. As wayfarers we cannot treat the qualitative aspects of life with such lordly indifference. In practice, we are bound to consider distances in terms of the effort it will require to traverse them. We are now interested, not in the surface of the earth *en toto* but in the particular stations we must reach in our wayfaring. The scale of miles we made as geographers will show the same numbers of miles as we shall find in traveling. The great advantage of the scale is that it will enable us to traverse the space with an amount of fatigue so small that we may fail to see that it is a substitute miniature for the real experience. Thought, therefore, does not save us entirely from the "trial and error method" of lower organisms. With humanity, the trial and error method has been transferred to *thought* instead of action; and in a sense, thought is life in miniature, not a quantitative miniature, but a qualitative one, if the phrase may be permitted. The scale of miles is a qualitative replica of the original because it is stored in the memory,

and its only disadvantage is that it is necessary to remember when the time for action arrives that each number shall have gradually restored to it, to the very end of the journey, the full measure of quality taken from it during the computation. This full measure must be returned in a varying proportion of *intension of effort* and of *extension of time*. During the journey we may save effort by sacrificing more time, and time may be saved by the intension of the effort: but a complete reduction of one to the other is inconceivable. What is the role played by time and effort during the period of calculation? We shall see that here there is a transposition of the factors involved, where there is an intension or contraction of time and *an extreme extension of effort*. But here, as before, there can be no complete reduction of the one to the other. *In other words, there can be no complete elimination of effort (and, therefore, of time) out of computations, any more than there can be a complete elimination of space out of action.* So inexorably are both facts of experience that a monistic philosophy could only gain credence by eliminating either the one or the other from experience; and the attempts and failures to do this are equally notorious in the history of thought. It is the business of thought, not to deny the experience of space or time but to show how they can concur in the same experience. What the nature of that concurrence is can already be vaguely seen: it is certainly functional whatever else it may be.

We may next concern ourselves about the necessary meaning of the term *absolute*. If it is not to encroach on the meaning of the word *relative* this term must hold that the signification of a thing depends on itself alone. As Bergson has it, a thing is absolute by being absolutely what it is. It is at once obvious that the term is misapplied when referred to a world of space whose points offer only relations. The most common misuse of the term is to take it

to refer to a thing in the aspect of its "wholeness" or "allness." We cannot say that a single atom or the whole universe of them are absolute for this would involve the meaningless comparison of a thing with itself, and comparison belongs to the relative and not to the absolute. The only aspect of our experience to which the term absolute may be legitimately applied is to *time*, for *time alone is undivided and this alone forbids relations.* Our very inability to conceive limits to the largeness of the universe or the smallness of it arises because we take it to be a universe of space, and space by its very definition is infinitely divisible or extensible; hence, infinitely relative. It will be objected that successive intervals of time may be related and compared, but in truth such comparison are always made of time already flown and quantified. When we say that the time now passing is more intense or less intense than heretofore we take advantage of the dual signification of the term *intensity;* and it is necessary to rescue this word from equivocation. It has misled psychological speculation because it implies at once the ideas of quality and quantity. It has thus been illegimately employed as a means of measuring qualitative experiences, a goal that has long been dear, not to Weber and Fechner alone, but to the scientists of all ages too enamoured of mathematics. We quantify a sensation as a measurable intensity because we prefigure its result in action even while experiencing it. There is in every sensation a reflexion of the extensity which it must meet, but to endure in memory they must be purely qualities, and as such, they are not relative. Time in a relative sense has no experiential meaning.

However, time is an absolute in a quite different sense than the word was employed by Newton. In Newtonian physics time is considered an absolute in the sense of a uniform flow. In the literature of modern physical relativism, it is common to refer to the Newtonian conception

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of time as "the relativism of Newton" to distinguish it from the "relativism of Einstein." The relativism of Newton is supposed to inhere in the inference that since time is uniform in its flow, its actual rate, like the actual extent of distances is negligible. This form of relativism is supposed to be distinct from Einstein's in that the latter considered the flow of time as relative to bodies while Newton considered its flow independent of them. Examination will show that the difference is only a verbal one. Underlying both conceptions is the notion that time is a literal flow, like that of water. Now, it is only a verbal difference whether we say that this flow is relative to bodies, as does Einstein, or that the motion of bodies is relative to the flow of time, as does Newton. If we once say, as does Newtonian physics, that time is absolute, we cannot then say that its *absolute rate* does not matter, for we do so only by smuggling in some other absolute by means of which its rate is determined. Such a conception of time should perhaps be described as "relatively absolute" and would not be one whit different from the time of Einstein which is "absolutely relative." Both of them, De Morgan would surely say, go beyond all serious paradoxing, and belong in the class with "round squares" and "square circles." Clerk Maxwell is the author of that other paradox which holds that should all bodies at the same time receive a blow that would increase their motion by a proportionate amount, the change would go undiscovered. Poincare has invented a similar one in which it is held that should the universe expand and contract uniformly, man would not be able to discover it. In either case, the reason why the changes are not discovered is because they are supposed to occur *instantaneously*, that is, without the change from one state to the other occupying an actual duration. Now, an instantaneous change is as bad a paradox as any. If no time is permitted for the "changes" of which they speak, then indeed, no change has

occurred. The seeming paradoxes arise because we imagine ourselves at once to be subject to the change, but that we still carry with us the memories of the previous universe.

Let us apply some of these considerations to the details of the physical conceptions of Newton and Einstein. Both seek an eternally conserved universe, and this is one, as we have seen, that must be unaffected by time. All changes are to be purely quantitative, and the vehicle in which they occur is the ether. Not only this, but ether is intended to solve that other persistent problem of the problem as to how in a timeless universe there can be an exchange of influences between mutually external objects. To illustrate, we will allow A and B to represent any objects in juxtaposition, two electrons, or, if you please, two stars. How is one to impart its motion to the other? A cannot displace B until B moves, and B will not move until A moves it. Not only are we left in this deadlock but the whole concept of contact between bodies is hardly imaginable. We can only say negatively that two indivisible atoms are in contact when they are neither separated nor invading. The miracle of transmitting the motion of one to the other occurs on contact, for manifestly that transfer could not occur while they are completely separated. But how are we to conceive the contact to accomplish the transfer? If the contact is a passive one it will not do so. On the other hand, how can a contact be active without involving at once the two contradictory notions of a simultaneous separation and invasion? Such a contact would moreover have to be instantaneous, which, again, leaves it merely passive. But physics has found in the concept of elasticity an escape from the difficulty. The property of elasticity will allow B to yield to A and we are able to image the rebound of B. But since both A and B are by hypothesis indivisible, we cannot say that they become distorted for

this supposes that they are not indivisible but are themselves compositions of parts into which distortions may enter. The problem of interaction would thus not be solved by the concept of elasticity but only transfers it to other *A's* and *B's* within the first *A* and *B* with which we began. Thus, when we logically follow the simple impacts of objects, constantly witnessed by the eye, we are left in a mystifying quandary. The difficulty we have hit upon is, stating it generally, the difficulty of conceiving how there can be a change of one state to another without an intermediate time evolution. This factor atomism alone cannot provide, so the physics of Newton has invoked ether to supply the defect. We shall now inquire whether the ether theory unties the Gordian knot or only cuts it.

The ether is the medium of exchange when energy is sent from body to body; and during the transmission, serves as the storehouse of potential energy. In other words, it is a substitute for an evolution in time without which a change from one state to another is unthinkable. The theory of ether is therefore at once used by science to embody the contradictory notions of an eternally conserved universe and an eternally evolving one. By this means science is enabled to regard its utterances as not only *useful* but as *true* in the philosophical sense. In short, atomism would be at once the description and the explanation of experience. But we have already seen that a harmony between the idea of the reality of development and the idea of the unreality of time is impossible; and science has managed to secure the appearance of such a synthesis by shifting the incongruity to the concept of the ether. Ether, accordingly, embodies a group of contradictory properties: it is at once matter and non-matter; it exists as do other bodies but offers no resistance; it is imponderable but still transmits energy in the form of waves as real as those of an unquiet sea; it is at once perfectly rigid and perfectly

elastic. That Michaelson, Morley, Miller and others should fail to find objective evidence of the existence of such an ether is hardly surprising! The great weakness of Newtonian physics is that it fails to provide an image of how energies are transferred in an eternally conserved universe. But not only has it failed here, but further inquiry will show that its failure is completed by its failure to image the universe as at all eternally conserved.

In Newton's view the universe is infinitely extended and the result is that when we wish to determine the gravitational effects at any point within it we are unable to arrive at a final sum, for these effects will not terminate in a given amount since they are eternally added to from gravitational effects coming from the ever deepening removes of space. The only solution for this difficulty is to suppose that our universe is finite and measurable. But this offers difficulties as great as the last for now we are again without a quantitative universe for its average density will approach zero. The universe will in this view be eventually dissipated by radiation in straight lines into infinite space and would eventually cease to exist, a result far from the doctrine of conservation.

The failure of Newtonian physics offers cues to the relativists for a better solution. To save the situation it is necessary to invent a scheme of things giving us both the properties of space and time without actually admitting the latter as in the nature of a growth or maturation. There is necessary an image of the universe in which it will appear as eternally conserved but still with a history. Now, to have a natural history it is necessary to provide against the possibility of a stable equilibrium. A stable equilibrium, however, is inevitable to a finite universe or to any "closed system." Science must therefore suppose, in order to have a continued dynamism, that infinite recesses of space exist from which such influences come,

since it denies creation. On the other hand, to have an eternally conserved universe it is necessary to have one which is in some sense finite, given once for all. But its finiteness must not be in the sense of a universe surrounded by immeasurable gulfs of emptiness for such a universe is already unhappily attenuated to zero or inexorably approaching such an extinction. To save the situation, there is required an image of the universe in which it will appear both finite and infinite.

It was the genious of Einstein that has made the last plausible attempt at such an accomplishment. Let us briefly state his hypothesis of "spherical space" which has been hailed everywhere as astounding both in its logical simplicity and consistency. We must not confuse his conception of a spherical universe with the ancient notion of a ponderous sphere beset with the fixed stars or the spherical universe of Copernicus. The universe is spherical to Einstein in the sense that straight lines do not exist, but when followed up will invariably return to their starting point. The universe is composed of the sum total of such lines of force, moving in immense circular paths of which the solar expeditions undertaken to verify Einstein's hypotheses have discovered objective evidence. The world is thus at once complete in itself but yet without boundaries. It is limited but it still constitutes the entire universe. In short, we have here a universe that is apparently at once finite and infinite; infinite in its dynamic aspects and finite in its amount; it is a universe with four dimensions.

Before criticising Einstein's solution, let us restate what we shall require of it. It must succeed where Newtonian physics has failed, and this failure was two-fold. It failed to show how mutually external forces can interact, and it failed to show how forces can be conserved.

Is the problem as to how A can impart its motion to B simplified if we suppose them to be moving in curvilinear

fashion instead of in straight lines? There certainly will be no gain for this view if we conceive of circular motion as atomism conceives it. Atomism will either say (in the language of the calculus) that a circle is a succession of infinitely short lines constantly changing their direction, or that it is a row of points behaving in the same way. Since lines cannot be infinitely short we must conclude that at the instant of impact between A and B they are moving in rectilinear fashion and in this case we meet with all the objections confronted in Newton's view. Since points cannot be conceived as real without possessing diameters, they are equally inadequate. Einstein's universe, therefore is constituted of an infinite number of circles as mutually external to each other as the points in the Newtonian scheme. In both systems, each circle or point is a universe by itself without any relation to the others except it be in the mind of some onlooker.

In the last statement we have the answer also to the question as whether Einstein's universe is a conserved one. It may be said of both the points in Newton's system and the geodetic lines of relativism that neither are conserved since their conservation depends on a relation, and unless that relation is itself a generative one it must be a relation that is *understood* by an onlooker. *Thus the universe to be preserved must first be preserved in time; it must have a duration of its own.* If we set aside the logical difficulty of interaction in Einstein's universe, and allow the curvilinear forces therein to interact they will at once become involved in an interpenetration during which every force will modify every other and will itself be modified by all just as a falling pebble in Newton's system will jar the entire universe and will express in its reaction the momentum that the universe possesses. In Einstein's scheme, the total effect would be as that of a sphere which rotates at once in every conceivable direction, an impossible image!

Such a universe would be essentially a living one, before which all images fail. If the number of his forces were finite they would end eventually in a completely homogenous stability; and would be none other than Newton's absolute space, in short, would be non-existence. Einstein only saves the continuation of his scheme by supposing the number of his forces as infinite in number: for Newton's infinite space he substitutes infinite relations. And just as Newton's infinite space can be rendered intelligible only by Aristotle's view that it is pure possibility, a potter's clay for the creative spirit, so in Einstein's view the infinite relations are saved only by an onlooker who preserves them by remembering them. Aristotle held that matter arises when spirit impresses pure possibility with form, and Einstein's view will be inconsistent unless it holds that matter arises not when a relation is *seen* but when a relation is *achieved*. We are here in a position to see what space really signifies. It is because the spirit continually renews its achieving that old tasks are conceptualized as the points of an imaginary space stretched beneath them. To the spirit every form it discerns is an instrument of its freedom; and the ideal of life were to make of the entire universe a simple lever that will yield too rather than intercept the will. Every form is a worth only because it enhances life; and for it to become of greater worth it must enter with other forms into a higher construction, a more specialized instrument of freedom. This means that in order to become a higher form any given form must for the sake of calculation become a pure possibility else calculation will be crippled by being limited. Hence it must become a point in an ideal space retaining of its former character only its quantitative aspect. In the idea of space is at once the idea of utter barrenness and of infinite possibility, and of these simple experience gives us no wisdom. It is matter alone, embodying at once a promise and an

achievement that is thus able to mirror the moral discovered in daily life that though life is irreversible, the choice of it is reversible. Space, therefore, is time stratified. *It is a momentary laying side by side of the many victories of life so that a major achievement may be projected from them.* It is the function of perception to cut the surrounding world into unit objects, each of which have been partly but not wholly subjected to the human will. The properties of space which they possess express the extent of their submission; and were they purely spacial they would indeed be no more than ideal, and the universe would spring miraculously into existence at the behests of thought: it would be a universe completely conquered, completely passive: it would be the pure space, the pure possibility of Aristotle. But *real* objects possess other properties than that of pure space—the property of resistance. As we have seen, resistance cannot obtain without time in which it occurs. *It is in this direction that interactions may be solved—by seeking to understand the very experience in which they occur, the experience of strife.* It is in this way that the inert must be explained by the living, and not the living by the inert. Relativism may mark another mile-stone in physics apparently because it has saved the principle of conservation. But what is the inner meaning of this achievement? It is, that relativism marks an epoch in the history of science because it saves the creative power of science by thus saving the possibility of prediction. That is, relativism is destined to be a tool in the hands of a creative force in the universe it so expressly denies. Nay, relativism was itself born of the creative imagination of man.

C. O. WEBER.

UNIVERSITY OF NEBRASKA.

CRITICISMS AND DISCUSSIONS

THE REALISM OF TONGIORGI

IN THESE DAYS of interest in what is called the New Realism, and its dealings with Being as wider than Reality, it is of interest to recall the positions of that distinguished thinker of last century, Tongiorgi on some of the issues now engaging attention. This particular purpose is the reason for my not discussing certain matters in Tongiorgi, such as the *sensus fundamentalis* described by him and by Rosmini, and critically reviewed by certain scholastic writers of distinction. The moderate-minded Scholasticism of Tongiorgi gives him the greater claim on our attention.

In his "Institutiones Philosophicae" (3 vols., Rome, 1861), Tongiorgi says: "Whatever is conceived as having some reality (*aliquam realitatem*), is conceived either as actually existing, or at least after the manner of something existing; for it is such that there is in it no repugnance to existence. Nay, more, by the very fact of its being an object of thought, it has a certain existence"—of the ideal or 'intentional' order, as he goes on to explain (Vol. II, *Ontol*, p. 8). Further on, he says: "Whatever has some reality, by which it can be an object of thought, has it either as existing, extra-mental (*extra mentem*) reality, or as not existing, but capable of becoming existent." This latter is *en in potentia* or *possible*, the former is *ens in actu* or *existens* (p. 23). This discrimination of ideal being is not really new, as often supposed, having had explicit recognition at least as far back as the time of Wyclif and Occam; neither is it, I would add, a discrimination peculiar to Scholastic philosophers, but made by many other philosophers, especially, but not exclusively, by those of realistic turn. They have held that whatever thought must be thought as though it had being or existence, and that possible being must be thought, as it would be, if it existed. That is to say, it must be thought under the attribute of existence.

Even Bradley has spoken of what "is real either inside of our heads or outside of them. And thus it always stands for exists." But I must leave others aside, as I have elected to speak of Tongiorgi.

Of the wide comprehension of Being opened by the recognition of ideal being, Tongiorgi says the *ens in potentia* or possible essence is privative in character, and belongs to the intelligible and metaphysical order, not to the order physical and actual. As later realists would say, it subsists, not exists. Possibility is, to Tongiorgi, "intrinsic" or "extrinsic." Intrinsic possibility is where there exists no repugnance to being in the constitutive nature of the thing, as, e. g., the possibility of a statue of gold. Intrinsic impossibility means a contradiction, that is, in its constitution, as, e. g., a quadrilateral triangle. Extrinsic possibility exists where there is fit or suitable cause to produce the thing, as, e. g., the possibility of a statue by an artificer. Intrinsic possibility is absolute, having no limit but contradiction; extrinsic possibility is relative. "Adequate" possibility embraces both (II, *Ontol,* p. 27). Elsewhere, Tongiorgi says that intrinsic repugnance means metaphysical impossibility (Vol. I, *Log.,* p. 225). Thus the unreal becomes reduced to what is intrinsically impossible, or to what has no existence outside our minds, for which latter he claims, as we have seen, a certain ideal being as an object of thought. As present day realists would say, it has being, not existence, as being timeless. Tongiorgi's discussion of possibility is in keeping with the unwonted, but merited, attention devoted to this subject by many Scholastic philosophers. Not that appreciation of the category of possibility has been confined to the Scholastics, for others, like Leibniz and Weisse, have made high use of possibility. Thus Bertrand Russell speaks of philosophy as "the science of the possible," as an "inventory of possibilities," and as a "repertory of abstractly tenable hypotheses." But is not this, it might naturally be asked, to assume for the knowledge of abstractions a higher dignity than for knowledge of the world in which we live? Is it not to assume an all too complete independence for purely conceptual entities and thought-possibilities? The concept is not a quid or simple entity which can be apprehended immediately outside us. in the mode assumed, without any need of involved mental process in the unity of a consciousness. Consciousness is not an aggregate of facts that are only exteriorly connected. The concrete unity of the conscious subject is involved in every act of thought, however simple. Can we so easily discard or shuffle

off the world of experience for a hypothetical world of ideas, conceived as in complete independence of the former? It has been the precise and peculiar claim of moderate realism—which strikes me as having been marked by great good sense in realisms of the past—that it keeps the most abstract metaphysical speculation in wholesome contact with the actual world in which we live and move and have our being. Being, not possibility, was the primal idea to Tongiorgi, who maintained that the notion of possibility is an efflorescence of our knowledge of things existing.

I may here notice Tongiorgi's discussion of the objective reality of ideas. To him the first idea is that of entity. First principles are formed from this idea. From these first principles ideas are, in upspringing of experience, acquired or formed. The objective reality of the ideas either immediately follows, or, can at least he thinks, be demonstrated. His position as to the objective reality of ideas is that a thing, which is an object of the mind, is an entity (*aliquod ens.* I, Log., p. 278). For what, he asks, is the *ratio* of entity, if not thing and object? He thinks that if ideas lacked this objective value (*objectivus idearum valor*), the first principles also would be inane. Such a theory of ideas is a long anticipation of certain recent theories; it differs, equally with Russell's, from Plato's theory of ideas, which Russell calls "one of the most successful attempts hitherto made," in that it posits no simple reminiscence of the ideas, but a direct apprehension of them.

Relation, Tongiorgi distinguishes as real, and as logical. He thinks real relation exists between things independently of the thinking or comparing power of the intellect. He instances, in this exteriority of relations, the relation of cause and effect. Logical relation is made by the intellect, as in the case of the relation of identity with itself. Of things logically identical, one can be affirmed of the other; but this affirmability does not obtain in the case of real identity. He thinks it the mark of real relations that they exist in the nature of things. The whole order, harmony and beauty of the world exist for him as real relations. When things are really identical. he says, that means they are really relative, are really opposed, and so far are really distinguished. Identity differs, in his view, from unity, which is absolute. in that identity is relative, as supposing *plura* which, compared *inter se,* are reduced to a certain unity. In real identity, two things which are identical with a third according to a common concept, are identical *inter se* according to

this very concept. In logical identity, two concepts which are identical with a third according to the object, are identical *inter se* according to this very object. But I doubt whether Tongiorgi would have countenanced the tendency to erect the logical independence of two facts into their real separation. or to treat two terms and the relation between them as separate entities. Moderate realism, at any rate, has not regarded the formal element in relation—its *esse ad* —as a distinct and separate reality. Tongiorgi retains a very clear sense of whatever is *mera distinctio rationis.* It is one thing to distinguish, in the course of experience, forms and qualities, and another thing to erect these into separate entities. Tongiorgi holds, as objects of the intellect, not only the essences of substances, but the essences of the qualities of substances, adding that such qualities are *quaedam entia,* and have their own essence (III. *Psychol,* p. 162). The context shows that such matters as color, sound and figure, are what he has in mind. But although he takes *omne ens,* which is the object of intellect, in a wide sense, he goes on to say that, nevertheless, what is offered *"primo* ac *per se"* to the intellect, is, the essence of external objects or material things. He does not, however, wish acts of the knowing subject to be excluded, but points out that it is by the intellect these become object. Thus he differentiates. Distinguish as we may and must the various and diverse aspects of reality, we yet cannot separate or rend them asunder, but must hold them in the living synthesis of thought, the concrete unity of consciousness. This is the unity of reason, "quae universalium propria est, nec extra mentem invenitur in rebus, sed fit a ratione per cognitionem abstractivam et comparativam." (II, *Ontol.,* p. 45.)

Finally, as to the criterion of truth in Tongiorgi's realism. Truth is, to him, in true Scholastic fashion, an equation or the conformity of thought to thing. In a way that strongly reminds one of Aristotle, he takes evidence to be the criterion of truth.[1] Evidence, he says, may be immediate or mediate, may be absolute or hypothetical, for evidence does not pertain only to *à priori* truths. He explains that not ideas, but only judgments, are to be spoken of as evident, for ideas are not objects that we perceive. A judgment is called evident, in so far as it is objectively regarded. What is evident is true, he holds, precisely as certain other Scholastic philosophers have done. Evidence is taken to be the universal criterion of truth—

[1] Cf. my Discussion: "Aristotle and the Criterion of Truth," in THE MONIST, July, 1921.

atque ultimatum certitudinis motivum (I, Log., p. 361). He thinks this *quod est evidens* better than the *quod clare et distincte* of Descartes, with its vagueness and indefiniteness. Error, Tongiorgi thinks, is in the judgment; not in the senses, nor in intellectual apprehension. Falsity is the object of error—falsity under the guise or appearance of truth. He follows that notable metaphysician Suarez, in thinking that the intellect is not necessarily determined in its judgment, save by the evidence, and evidence cannot obey false judgment. The intellect can only be so determined by the will, when it is not free. Hence, he holds that every error has its origin in a free movement of the will. He thus puts a severe strain or responsibility upon the will. This idea has not been wholly absent from recent discussions on error, but it is safe to say that, neither in criticism, nor in furtherance of it, has this line of thought been so fully worked out as it could very well be.

JAMES LINDSAY.

IRVINE, SCOTLAND.

THE FALLACY OF EXCLUSIVE SCIENTIFIC METHODOLOGY

BY THE *fallacy of exclusive scientific methodology* I mean the claim that science possesses the only valid method of knowledge, together with the denial or at least the ignoring of the existence of that which is incapable of being studied by the method of science. This is a conspicuous fallacy in the thinking of many at the present time who are devotees of scientific method to the exclusion of any other means of knowledge, and who fail to recognize that there are limitations to the scope of the sciences. In the field of psychology the fallacy is prominent in the work of the extreme behaviorists. It occurs also in the attempt of some to make scientific method the method of philosophy, which means the limitation of philosophy to such problems as are accessible to scientific investigation, with a denial, implicit at least, of the legitimacy of any other problems. Philosophers have been called "lords of the uncleared ground" of knowledge, concerned merely with matters that have not yet been dealt with by the sciences, and forced to content themselves with narrower domains after each new scientific conquest. Underlying such a conception of the unenviable position of philosophers there have lurked the ambition and the expectation of science eventually to dispossess philosophy completely by usurping the whole estate of learning. Were scientific method capable of dealing with all humanly significant problems, such a result would be inevitable and indeed "a consummation devoutly to be wished." I propose, however, to point out that there are problems which by their very nature are incapable of being reached by the scientific method and which, therefore, will remain indefinitely as distinct philosophical questions, questions, moreover, the importance of which is not surpassed by that of any of the problems of science.

In order to understand the limitations to the scientific method, it will be necessary first to state briefly what this method is. Its chief characteristics are observation, experimentation, and the application of mathematics in the formulation of the laws which observation and experimentation bring to light. It was Bacon who first clearly sounded the call to exact observation—to an accurate read-

ing of the book of nature without any prejudiced anticipation of what its contents might be. Experimentation is simply observation of events carried out under conditions of control such that the events may be isolated and repeated under identical or varied conditions, as the experimenter's wish may be. Then the conclusions of observation and experimentation are not only arranged in orderly form but they are given mathematical expression so far as this is possible. Kant said that a body of knowledge is scientific only to the extent to which it may be cast in mathematical form. Though there are other factors involved in scientific procedure, and though common usage of the term "science" justifies its application in fields that are not so mathematically exact as, for example, physics or chemistry, there is unanimity of emphasis in all the sciences upon observation of facts as the basis. As Professor Titchener says, "Scientific method may be summed up in the single word, 'observation.'" (*A Text-Book in Psychology*, p. 19.)

Scientific observation implies the existence of objects and the occurrence of events which are capable of being seen, weighed, and measured, or at least of being inferred from their sensible effects, not merely by one observer but by the whole body of scientific investigators who may take the trouble to examine the facts in question. As Professor Royce has said, "Successful description, made with any scientific purpose, seems to involve the possibility of comparing together the various attempts at description made by different observers in view of the same facts." (*Outlines of Psychology*, p. 5.)

The behaviorists deserve credit for having recognized and applied this principle. Their criticism of introspection as a scientific method and of introspective psychology as a branch of natural science is wholly justified. Mr. Watson has clearly stated the case against introspective psychology as a science in the following sentences (*Behavior*, pp. 6, 26, 27): "Psychology has failed signally during the fifty odd years of its existence as an experimental discipline to make its place in the world as an undisputed natural science." "It has enmeshed itself in a series of speculative questions which . . . are not open to experimental treatment." Mr. Watson and other behaviorists are determined that their science shall be really scientific, based solidly upon observation and experimentation, without resort to introspection. For the first time in history, in the hands of Mr. Watson and other behaviorists, psychology (for behaviorism still retains this name) has become a genuine science. As

Mr. Watson says: "The key which will unlock the door of any other scientific structure will unlock the door of psychology [i. e., of behaviorism]. The differences among the various sciences now are only those necessitated by the division of labor. Until psychology recognizes this and discards everything which cannot be stated in the universal terms of science, she does not deserve her place in the sun. Behavior psychology does make the attempt for the first time" (*Psychology from the Standpoint of a Behaviorist,* p. vii).

By what sleight-of-hand performance, however, it may be asked, has non-scientific psychology transformed itself into scientific behaviorism? It has done so by ceasing to be a study of consciousness, which requires introspection, a non-scientific procedure, and by becoming merely a study of behavior, which is indeed open to strictly scientific observation and experimentation. Thus Mr. Watson, in criticizing introspective psychology, speaks of the "mistaken notion that its field of facts are conscious phenomena and that introspection is the only direct method of ascertaining these facts" (*Behavior,* p. 26). He says (*Behavior,* pp. 7, 9): "The time seems to have come when psychology must discard all reference to consciousness; when it need no longer delude itself into thinking that it is making mental states the object of observation." "It is possible to write a psychology, to define it . . . as the 'science of behavior,' and never to use the terms consciousness, mental states, mind, content, will, imagery, and the like." And in his latest book (*Psychology from the Standpoint of a Behaviorist,* p. viii), he says, "the reader will find no discussion of consciousness."

If Mr. Watson and other behaviorists of his type did not continue to apply the term "psychology" to their science, few people would have any quarrel with them. We would agree that introspection is not a scientific method; we would agree that consciousness cannot be studied otherwise than through introspection; and we would therefore agree that the study of consciousness cannot become a science. We are justified in objecting, however, to the application of the term "psychology" (which means the study of consciousness if it means anything) to the new science when defined explicitly as being *not* a science of consciousness. And we are justified still more in objecting to the tendency of some behaviorists to deny the existence of consciousness from the fact that it is incapable of being studied by the method of science. In some of the above quotations from Mr. Watson consciousness is recognized as a fact, but as a

fact to be ignored since it is outside the possible field of scientific investigation. In the following passage, however, Mr. Watson asserts his faith that sometime behaviorism will study even consciousness itself. This would mean an implicit denial of the existence of consciousness in itse essential and unique nature. "Psychology as behavior," Mr. Watson says, "will, after all, have to neglect but few of the really essential problems with which psychology as an introspective science now concerns itself. In all probability even this residue of problems may be phrased in such a way that refined methods in behavior (which certainly must come) will lead to their solution" (*Behavior,* p. 28). To say that the observational method of behaviorism can eventually solve the problems of introspective psychology is to deny by implication the existence of consciousness, since consciousness, as has been indicated, and as will be shown more fully below, is something which can never be studied as an object among objects by the scientific method. Mr. Watson and other extreme behaviorists are guilty of what I have called the fallacy of *exclusive scientific methodology.*

My position will be made clearer by a further discussion of the meaning of consciousness and of the reason why it cannot be studied by the scientific method. So far as objects in the outside world are concerned, including other persons, there is no reality of which we are certain except such as is capable of common observation. We observe the behavior of animals and of men, but we are unable to *prove* that they are conscious. Even if our powers of observation were infinitely magnified so that the activity of each brain cell in a person under observation were capable of being inspected, we should observe nothing except what is the subject-matter of physiology and, in the last analysis, of physics and chemistry. As Professor Paulsen has said: "Let us imagine with Leibniz the skull of an animal or man to be as large as a mill. Suppose one could walk around in it and observe the processes of the brain as one can observe the movements of the machinery and the cogging of the wheels in the mill. . . . One would see as little of psychical processes, of ideas and thoughts, as in the movements of the mill

(*Introduction to Philosophy*, p. 84). Does this fact, however, that scientific observation is limited to physical processes prove that there is nothing except the physical in existence? On the contrary, the existence of consciousness is proved in the experience of each individual by the fact of his own awareness. As Professor Royce has expressed it: "Were physiologists better endowed with sense organs and with instruments of exact observation, we can, if we choose, conceive them as, by some unknown device, coming to watch the very molecules of our brains; but we cannot conceive them, in any possible case, as observing from without our pains or our thoughts in the sense in which physical facts are observable. . . . No microscope could conceivably reveal them. To me alone, would these states be known. And I should not see them from without; I should simply *find* them, or *be aware* of them. And what it is to find them, or to be aware of them, I alone can tell myself" (*Outlines of Psychology*, pp. 4, 5).

Thus, whoever asserts that scientific observation is capable of studying all of what properly goes under the name of psychology is refuted by the experience of each individual. There is consciousness and it is known directly only through introspection, which is not a scientific method inasmuch as its objects are not objects of common observation. What I refer to in saying, "I am conscious," or "I experience a sensation," is beyond the reach of a purely objective scientific study. To assert that scientific method is the only method, and especially to assert that nothing exists which scientific observation is incapable of reaching, is to commit the *fallacy of exclusive scientific methodology*.

The objection might be raised, however, that consciousness may be capable of becoming an object of scientific investigation through its sensible effects, just as in the case, for example, of electricity. Scientific hypotheses regarding electricity may be tested through observation of its sensible effects. There is not a correct analogy here, however. Few of those who assert that consciousness is a fact would admit that it has any observable effects in the outer, physical world. To assert that it does influence physical events

would be to affirm the theory of psycho-physical interaction, which is contrary to the principle of the conservation of energy. The existence of electricity and of similar scientific entities "makes a difference" in the occurrence of observable external events, but consciousness, in the sense in which the term is employed in this article, cannot "make a difference" in the phenomena of physics and of physiology.

Behaviorism may limit itself to an objective study of behavior merely, without using terms referring to psychical contents, while admitting that consciousness is real but accessible only to a philosophical study; and when it does this I classify myself as a behaviorist. It seems to me probable that there is a mechanistic, i. e., physico-chemical, basis of all human behavior; including language and the processes involved in such complex responses, for example, as those of Shakespeare in writing his plays or in the work of a mathematician in formulating the principles of mathematics. And I am in hearty accord with the general spirit of the work of such a biologist as Mr. Jacques Loeb in his studies of behavior. Mr. Loeb, however, commits the *fallacy of exclusive scientific methodology* when he comes to a discussion of consciousness. Objecting to the term "consciousness," which he correctly calls a metaphysical concept, he substitutes the term "associative memory." (See *Physiology of the Brain*, pp. 214, 15, 17, 32; *The Mechanistic Conception of Life*, p. 73.)

He then proceeds to define associative memory" in purely objective terms, as docility, or the capacity of the organism to learn new responses and consequently to modify inherited forms of response. He says, for example: "By associative memory I mean that mechanism by which a stimulus brings about not only the effects which its nature and the specific structure of the irritable organs calls for, but by which it brings about also the effects of other stimuli which formerly acted upon the organism almost or quite simultaneously with the stimulus in question" (*The Mechanistic Conception of Life*, pp. 73, 74). This is a case merely of the "conditioned reflex," and in his most recent book Mr. Loeb employs the latter term

(*Forced Movements, Tropisms, and Animal Conduct,* p. 167). The fallacy involved in substituting the term "associative memory" for the term "consciousness," and then in defining associative memory purely in terms of behavior, is that of denying by implication the existence of consciousness in its proper sense, as a fact of inner experience inaccessible to a scientific study.

There is a philosophical type of behaviorism which, if its metaphysical basis could be proved correct, would study consciousness itself objectively, thus escaping the fallacy of ignoring or of denying the existence of consciousness. I refer to such a metaphysical theory of consciousness as that, for example, which William James has so well expressed in his philosophy of radical empiricism, and which has been incorporated in the philosophy of American neorealism. If one goes beyond both "common sense" and science, and asserts that the content of each personal stream of consciousness is identical with that portion of the outer environment to which the organism reacts selectively, then one may say that consciousness itself, conceived in the manner that such a theory presupposes, becomes an object of common observation. James, for example, maintained that "a given undivided portion of experience, taken in one context of associates, plays the part . . . of a state of mind, of 'consciousness'; while in a different context the same undivided bit of experience plays the part of a thing known, of an objective 'content'" (*Essays in Radical Empiricism,* pp. 9, 10). On this assumption consciousness itself may become an object of common observation, but this is a metaphysical assumption that no behaviorist merely as such can accept; nor can scientific method establish the truth of such an assumption.

In the field of general philosophy the *fallacy of exclusive scientific methodology* is committed by those naturalistic philosophers who claim for reality at large what extreme behaviorism claims for its special portion of reality, namely, the exclusive validity of the scientific method and the denial or the ignoring of the existence of anything beyond the reach of this method. A materialistic philosophy consisting of a generalization from the sciences may be refuted in

the same way as that in which an extreme behaviorism may be refuted. My personal awareness is, as has been shown, beyond the reach of scientific observation, and yet it is the thing that is most indefeasibly real for me. I thus am certain of at least one bit of reality of which not only extreme behaviorism but also a general philosophy of materialism, based exclusively on scientific method, denies the existence.

Whether or not some sort of an idealistic system of philosophy may be built up from this initial certainty of individual consciousness is another question, of which I will merely suggest a possible answer. The fact that man as viewed outwardly by the scientist is only a complex machine not different in kind from the other mechanisms in nature, while we acknowledge, though we do not observe, in every person a conscious life, which is purposive as purpose is experienced by ourselves—an inner freedom which is outwardly determined,—this fact suggests that the universe at large, though it is purely mechanical as the sciences describe it, no less so and no more so than the human organism, may also have an inner purpose, even a free and conscious purpose, no less real and perhaps infinitely more significant than finite human purposes. Just as I ascribe to other persons an inner, conscious life which objective observation does not reveal to me, while at the same time viewing them outwardly as mechanisms explainable in terms of stimulus and response, so to the totality of existence, explainable outwardly in terms of mechanistic science, I at least cannot deny the possibility of an inner life analogous to my own inner life and to that of my fellow men. To deny the possibility of such a reality on the ground that it is not an object observable by scientific method would be to commit the *fallacy of exclusive methodology.* If there be such a conscious purpose in the universe at large, it is by its very nature as inacessible to scientific observation as is my own consciousness.

It seems strange that even many philosophers should have been so much dominated by the prestige of modern science. That the popular mind should fall under the spell of science is easily enough

accounted for. Science is practical. As Bacon said, scientific knowledge is power over nature. As a result of modern science, nature has been turned to man's uses to an extent that earlier generations would not have believed possible. Applied science has yielded innumerable satisfactions to man's desires for physical health and comfort and achievement. The progressive conquest of nature by science even stirs the blood by its dramatic venturesomeness. In contrast with the picturesque story of modern science, the story of philosophy seems uninteresting to the type of mind that does not care to exercise its human prerogative of abstract thinking. Philosophy is not practical; it can bake no bread: and yet it deals with problems that are, in a sense, the most practical of all problems in so far as their solution can help mankind to feel at home in a universe that seems outwardly so alien to man's inmost interests.

While one can understand why the popular mind should think the term "scientific" always a complimentary one, and the term "philosophic" almost a reproach, one can hardly forgive this attitude in those of greater discernment. Why should the student of the mind object to the admission that a part of his field is not science but philosophy? Why not take the epithet, "arm-chair psychologist" as a genuine compliment (even though it be not intended as such)? The psychologist who does not go beyond the scientific method of the laboratory to the reflective method of philosophy fails to cover the whole field of psychology. Behavior can be studied scientifically and behaviorism can be applied in such practical fields as advertising, salesmanship, pedagogy, and the like. The scientific study of behavior can be of great assistance also, in an indirect way, to the understanding of purely theoretical problems of consciousness itself. The philosopher should make what use he can of scientific result. But psychology will always be in part a *philosophy* of consciousness or else it will be an incomplete psychology. The sciences may and ought to aim at a complete description of the physical world; and yet there will always remain, over and above the most complete scientific descriptions that are possible, philosophical problems such, for example, as the one briefly touched upon above. By its very

nature such a problem cannot be dealt with by the scientific method. The philosopher, if he rightly conceives of his problems and of his method, can never be displaced through the advances of science. He should co-operate with the scientists, welcoming each new scientific achievement, while being proud at the same time to be a worker in the important and distinctive field of philosophy.

WESLEY RAYMOND WELLS.

LAKE FOREST COLLEGE.